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ABSTRACT

The rationale behind the method of estimating the operating characteristics of discrete item responses when the test information of the Old Test is not constant was presented previously. In the present study, two subtests of the Old Test, i.e. Subtests 1, and 2, each of which has a different non-constant test information function, are used in place of the Old Test, and the item characteristic functions of ten hypothetical binary items are estimated on the simulated data of five hundred hypothetical examinees, using Simple Sum Procedure of the Conditional Probability Density Function (PDF) Approach and the Normal Approach Method. The research is designed to test the method as well as to examine the possibility of reducing the number of test items in the Old Test. The reduction of the number of test items from 35 in the original Old Test to 25 in both Subtests 1 and 2 proved to be acceptable. It was suggested from the results that even a smaller number of test items in the Old Test than 25 are probably needed. (Author/RL)

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RESEARCH REPORT 80-4

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ESTIMATION OF THE OPERATING CHARACTERISTICS WHEN THE TEST INFORMATION OF THE OLD TEST IS NOT CONSTANT II: SIMPLE SUM PROCEDURE OF THE CONDITIONAL P.D.F. APPROACH/NORMAL APPROACH METHOD USING THREE SUBTESTS OF THE OLD TEST

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The rationale behind the method of estimating the operating characteristics of discrete item responses when the test information of the Old Test is not constant was presented in one of the previous research reports, RR-80-2. In the present study, three subtests of the Old Test, i.e. Subtests 1, 2 and 3, each of which has a different non-constant test information function, are used in place of the Old Test, and the item characteristic functions of ten hypothetical binary items are estimated on the simulated data of five hundred hypothetical examinees, using the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method. The research is designed to test the method as well as to examine the possibility of reducing the number of test items in the Old Test.

ESTIMATION OF THE OPERATING CHARACTERISTICS WHEN THE TEST INFORMATION OF THE OLD TEST IS NOT CONSTANT II: SIMPLE SUM PROCEDURE OF THE CONDITIONAL P.D.F. APPROACH/NORMAL APPROACH METHOD USING THREE SUBTESTS OF THE OLD TEST

ABSTRACT

The rationale behind the method of estimating the operating characteristics of discrete item responses when the test information of the Old Test is not constant was presented in one of the previous research reports, RR-80-2.

In the series of research including the present study, several subtests of the Old Test, each of which has a different, non-constant test information function, are used in place of the Old Test, and the item characteristic functions of ten hypothetical binary items are estimated on the simulated data of five-hundred hypothetical examinees. In this paper, our initial intention was to use three subtests, i.e., Subtests 1, 2 and 3. Because of too large a number of pages for a single research report, however, the decision has been made later that a part of the initial plan has to be excluded and wait for another report. Unlike the title of the present paper, therefore, only two subtests of the original Old Test, i.e., Subtests 1 and 2, are used in the present paper. The combination of an approach and a method used in the present series of studies are the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method. The research is designed to test the method as well as to examine the possibility of reducing the number of test items in the Old Test.

The research was conducted at the principal investigator's laboratory, 409 Austin Peay Hall, Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked as assistants for this research include Paul S. Changas, Chi-Lin Tom, Pamela Welch, Dete Furlan, Linda Lance, Melanie Perkins, and Suzanne Wiseheart.

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I Introduction

The rationale behind the method of estimating the operating characteristics of discrete item responses when the amount of test information of the Old Test is not constant was introduced in a previous study (Samejima, 1980b). It is an expansion of the various combinations of a method and an approach which were developed and presented earlier and in which the test information function of the set of items, or Old Test, whose operating characteristics are known is constant for the interval of ability θ of our interest. The key process in the generalized method is to consider an intervening variable, τ, which is a strictly increasing transformation of θ and has a constant amount of test information for the entire interval of θ of our interest.

It should be noted that this expansion of the method has substantially broadened the applicability of the method, since it is rather difficult to develop a set of discrete response items which has a constant test information, especially in the paper-and-pencil testing situation. In computerized adaptive testing, however, this can be achieved more easily since we can adjust the test information locally, by using a specific amount of test information as the criterion for terminating the presentation of new items (cf. Samejima, 1977a, 1977b). Note that in such a case our Old Test is not a fixed set of test items, but various subsets of test items selected for specific individuals from the item pool.

In the present paper, we shall apply the generalized method for some simulated data, using two different types of test information functions, to find our how well the method works. In so doing, we choose the Simple Sum Procedure of the Conditional P.D.F. Approach combined with the Normal Approach Method (Samejima, 1978a, 1978b) as our example, and the results are compared with those previously obtained by using the original Old Test.

The present study also provides us with an opportunity to observe how the reduction in the number of items, or in the amount of test information, of the Old Test affects the accuracy of estimation of the operating characteristics of new items. This part of research is meaningful, since, if the loss in accuracy of estimation is negligibly small, the smaller number of items in the Old Test will definitely be preferrable.

It is noticed that the term "graded response" is replaced by "discrete responses" in the present paper. This has been done intentionally, since these combinations of a method and an approach are applicable not only for the estimation of the operating characteristics of graded item responses but also of those of any discrete item responses, including the operating characteristics of the distractors of the multiple-choice item (cf. Samejima, 1979, 1980a).

II Estimated Item Characteristic Functions of Ten Binary Test Items Obtained upon Subtests 1 and 2

In this chapter, we shall use two subtests of the original Old Test, which has a constant test information for the interval of [-3.0, 3.0] , and has been used repeatedly in the previous studies (Samejima, 1977c, 1977d, 1978a, 1978b, 1978c, 1978d, 1978e, 1978f). They are Subtests 1 and 2, which were introduced in a previous paper (Samejima, 1980b), and each of which contains twenty-five graded test items. The discrimination parameter, a_g , and the two difficulty parameters, b_1 and b_2 , of each of these items, which follows the normal ogive model (Samejima, 1969, 1972) such that

$$(2.1) \qquad P_{x_g}(\theta) = [2\pi]^{-1/2} \int_{a_g(\theta - b_{x_g+1})}^{a_g(\theta - b_{x_g})} e^{-u^2/2}\, du ,$$

where $P_{x_g}(\theta)$, is the operating characteristic of the item response x_g to item g , are given in Table 2-1. The item response information function, $I_{x_g}(\theta)$, is defined by

$$(2.2) \qquad I_{x_g}(\theta) = -\frac{\partial^2}{\partial\theta^2} \log P_{x_g}(\theta) ,$$

and the item information function, $I_g(\theta)$, is given as the conditional expectation of the item response information function, given θ , such that

$$(2.3) \qquad I_g(\theta) = \sum_{x_g=0}^{m_g} I_{x_g}(\theta)\, P_{x_g}(\theta) .$$

TABLE 2-1

Item Discrimination Parameter, a_g , and Two Item Difficulty Parameters, b_1 and b_2 , of Each Item in Subtests 1 and 2 .

Item g	a_g	b_1	b_2	Subtest 1	Subtest 2
1	1.8	-4.75	-3.75		x
2	1.9	-4.50	-3.50		x
3	2.0	-4.25	-3.25		x
4	1.5	-4.00	-3.00		x
5	1.6	-3.75	-2.75		x
6	1.4	-3.50	-2.50	x	x
7	1.9	-3.00	-2.00	x	x
8	1.8	-3.00	-2.00	x	x
9	1.6	-2.75	-1.75	x	x
10	2.0	-2.50	-1.50	x	x
11	1.5	-2.25	-1.25	x	x
12	1.7	-2.00	-1.00	x	x
13	1.5	-1.75	-0.75	x	
14	1.4	-1.50	-0.50	x	
15	2.0	-1.25	-0.25	x	
16	1.6	-1.00	0.00	x	
17	1.8	-0.75	0.25	x	
18	1.7	-0.50	0.50	x	
19	1.9	-0.25	0.75	x	
20	1.7	0.00	1.00	x	
21	1.5	0.25	1.25	x	
22	1.8	0.50	1.50	x	
23	1.4	0.75	1.75	x	x
24	1.9	1.00	2.00	x	x
25	2.0	1.25	2.25	x	x
26	1.6	1.50	2.50	x	x
27	1.7	1.75	2.75	x	x
28	1.4	2.00	3.00	x	x
29	1.9	2.25	3.25	x	x
30	1.6	2.50	3.50	x	x
31	1.5	2.75	3.75		x
32	1.7	3.00	4.00		x
33	1.8	3.25	4.25		x
34	2.0	3.50	4.50		x
35	1.4	3.75	4.75		x

The test information function, which is denoted by $I(\theta)$, can be written as the sum total of the item information functions, i.e.,

$$(2.4) \qquad I(\theta) = \sum_{g=1}^{n} I_g(\theta) \quad .$$

The square root of this test information function for each of Subtests 1 and 2 is drawn by a solid curve, in each of Figures 2-1 and 2-2. We can see that these two curves are substantially different from each other, and lower than the constant amount of test information for the Old Test, which is approximately 4.65 , and is also shown in each of Figures 2-1 and 2-2 as a solid line parallel to the abscissa. In the same figures, also presented by dotted curves are the polynomials of degree 7 which were fitted by the method of moments (Elderton and Johnson, 1969) to these two square roots of the test information functions, respectively. It has been shown (Samejima and Livingston, 1979) that each curve is the least squares solution for fitting a polynomial of degree 7 to the original function. The interval of θ used in the process of applying the method of moments is [-4.0, 4.0] in each case. These resulting polynomials are:

$$(2.5) \qquad 4.72922 + 0.10599\theta - 0.03771\theta^2 - 0.04152\theta^3 - 0.01160\theta^4 + 0.00447\theta^5 + 0.00005\theta^6 - 0.00014\theta^7$$

for Subtest 1, and

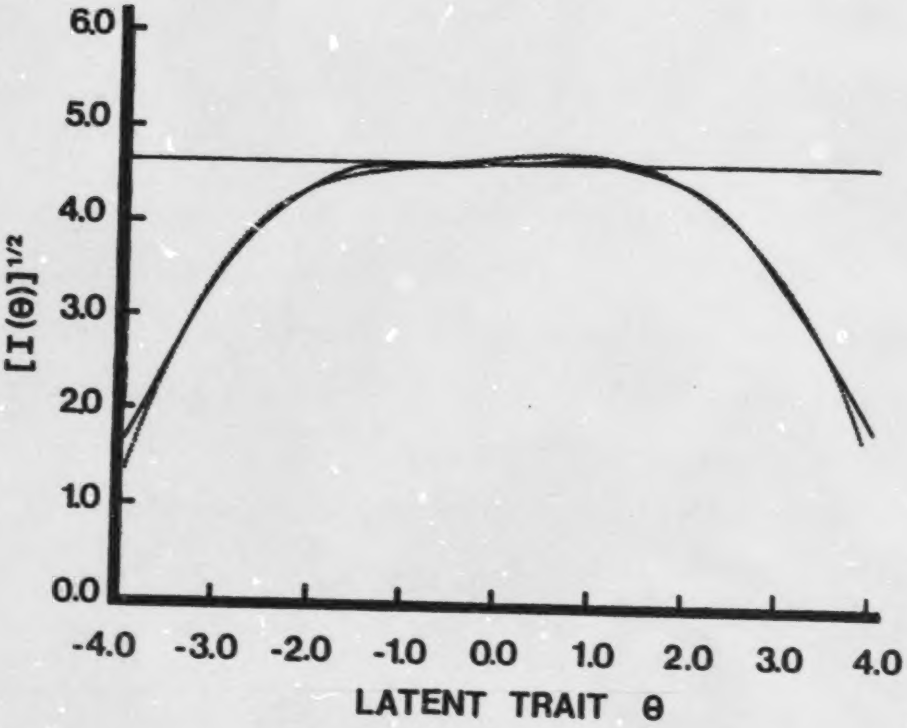


FIGURE 2-1

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) and the Polynomial of Degree 7 (Dotted Line), Which Was Fitted by the Method of Moments with [-4.0, 4.0] As the Interval of θ.

Subtest 1

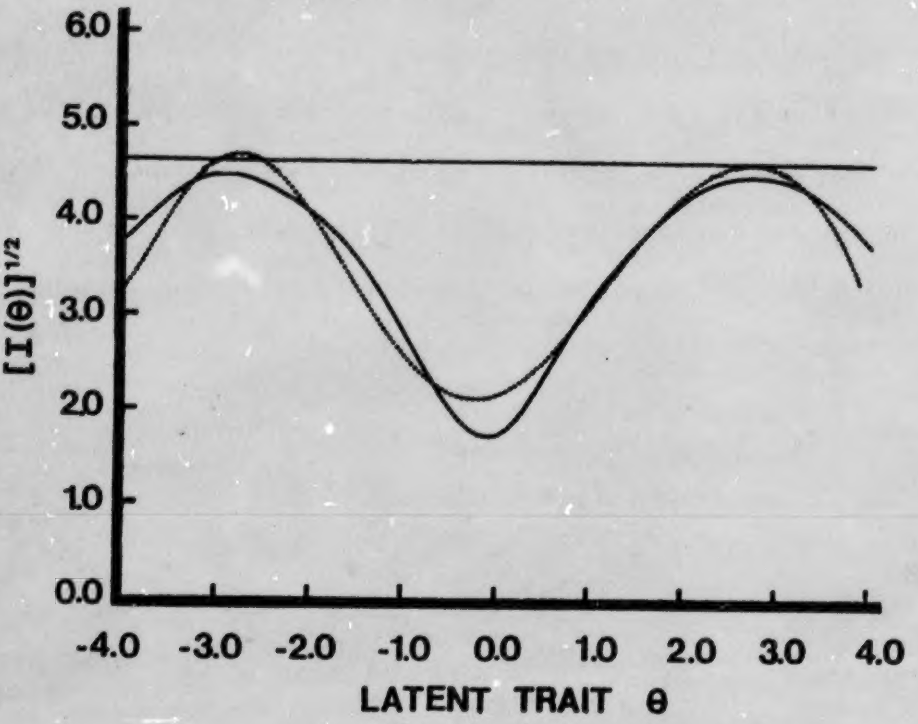


FIGURE 2-2

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) and the Polynomial of Degree 7 (Dotted Line), Which Was Fitted by the Method of Moments with [-4.0, 4.0] As the Interval of θ .

Subtest 2

$$(2.6) \quad 2.14813 + 0.37926\theta + 0.74646\theta^2 - 0.12415\theta^3 - 0.06554\theta^4 + 0.01252\theta^5 + 0.00143\theta^6 - 0.00040\theta^7 \quad ,$$

for Subtest 2.

Is has been shown (Samejima, 1980b) that the process of transforming θ to the intervening variable τ and the eventual retransformation of τ to θ will become much simpler if we use the approximated polynomial in place of the square root of the test information function itself. Let α_k $(k=0,1,\dots 7)$ be the coefficient in the polynomial given by (2.5) or (2.6). We can write for the transformation of θ to τ

$$(2.7) \qquad \tau = \sum_{k=0}^{m+1} \alpha_k^* \theta^k \quad ,$$

where

$$(2.8) \qquad \alpha_k^* \begin{cases} = d & \text{for } k = 0 \\ = (Ck)^{-1} \alpha_{k-1} & \text{for } k = 1,2,\dots,m+1 \end{cases} \quad ,$$

with d being an arbitrarily chosen constant and C the constant amount of test information for the transformed latent trait, τ. In our examples, zero is used for d, and $C = 4.5$ for Subtest 1 and $C = 3.5$ for Subtest 2. These coefficients, α_k^* $(k=0,1,\dots,8)$, are shown in Table 2-2 for each subtest.

Five hundred hypothetical examinees are used in the present study, whose ability levels are located at one hundred different points of θ, which start from -2.475 and end with

TABLE 2-2*

Coefficients α_k^* for the Transformation of θ to τ .

k	Subtest 1	Subtest 2
0	0.00000	0.00000
1	1.05094	0.61375
2	0.01178	0.05418
3	-0.00279	0.07109
4	-0.00231	-0.00887
5	-0.00052	-0.00375
6	0.00017	0.00060
7	0.00000	0.00006
8	-0.00000	-0.00001

---

* This table is missing in RR-80-2, and is erroneously referred to as Table 3-3, on page 44, in the second line from bottom, of RR-80-2.

2.475 with equal steps of 0.05 , and each of which accomodates five examinees. The response pattern, V , for each hypothetical examinee was calibrated by the Monte Carlo method for each subtest, and the maximum likelihood estimate, $\hat{\theta}$, was obtained as the solution of

$$(2.9) \qquad \sum_{x_g \varepsilon V} A_{x_g}(\theta) = 0 \quad ,$$

where $A_{x_g}(\theta)$ is the basic function for the item response x_g , which is defined by

$$(2.10) \qquad A_{x_g}(\theta) = \frac{\partial}{\partial\theta} \log P_{x_g}(\theta) \quad .$$

The other maximum likelihood estimate, $\hat{\tau}$, for the transformed latent trait τ is given by

$$(2.11) \qquad \hat{\tau} = \sum_{k=0}^{m+1} \alpha_k^* \hat{\theta} \quad ,$$

by virtue of the transformation-free character of the likelihood function.

Figure 2-3 presents the relative frequency distribution of the five hundred $\hat{\tau}$'s thus obtained upon Subtest 1 in the form of histograms. In these two graphs, the polynomials of degrees 3 and 4, which were obtained by the method of moments upon the set of these five hundred observations to provide us with approximations to the

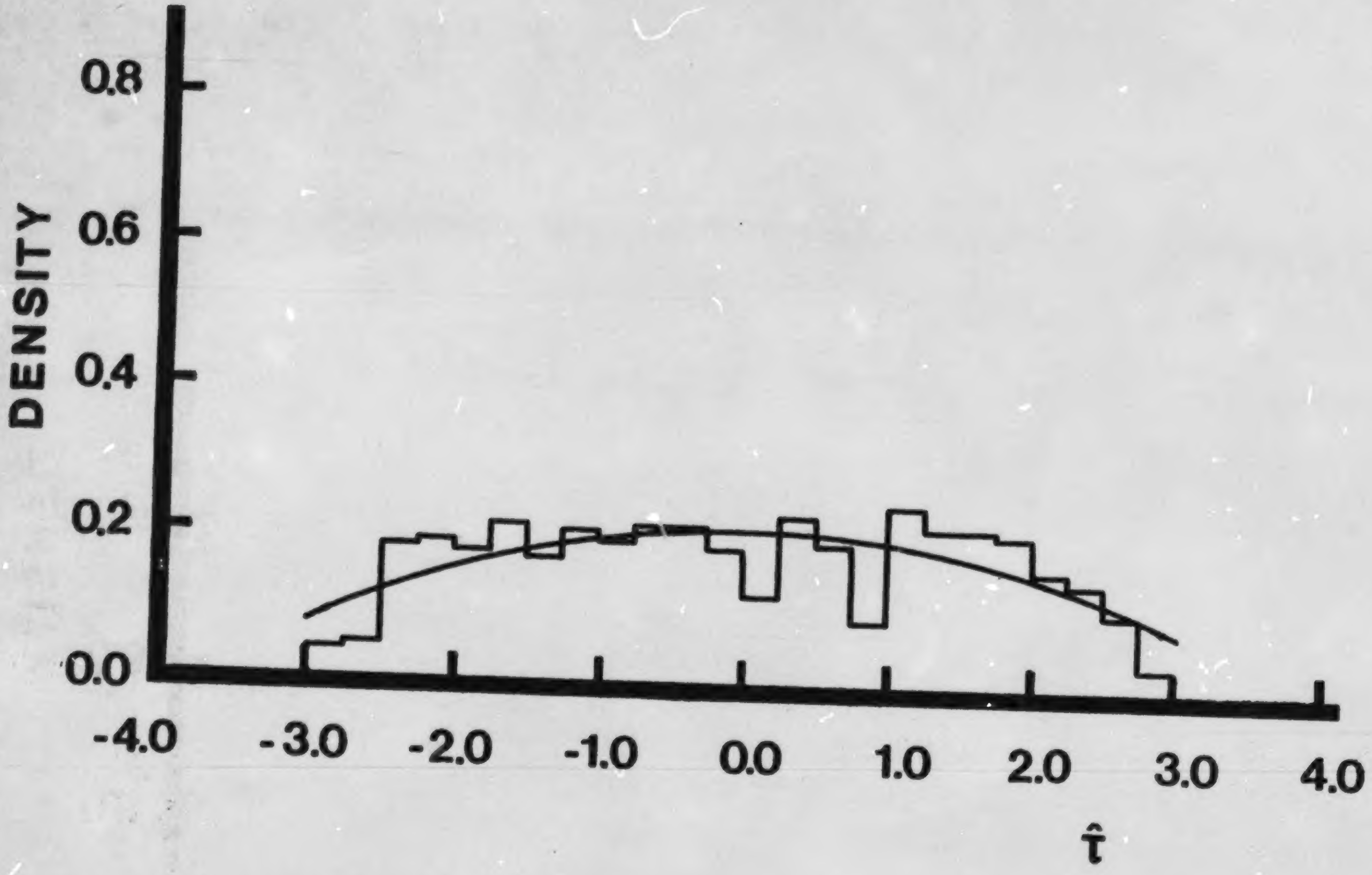


FIGURE 2-3

Relative Frequency Distribution of $\hat{\tau}$, Which Was Obtained for the Five Hundred Hypothetical Examinees on Subtest 1, with 0.25 as the Subinterval Width, Together with the Polynomial of Degree 3 Obtained by the Method of Moments to Approximate the Density Function of $\hat{\tau}$.

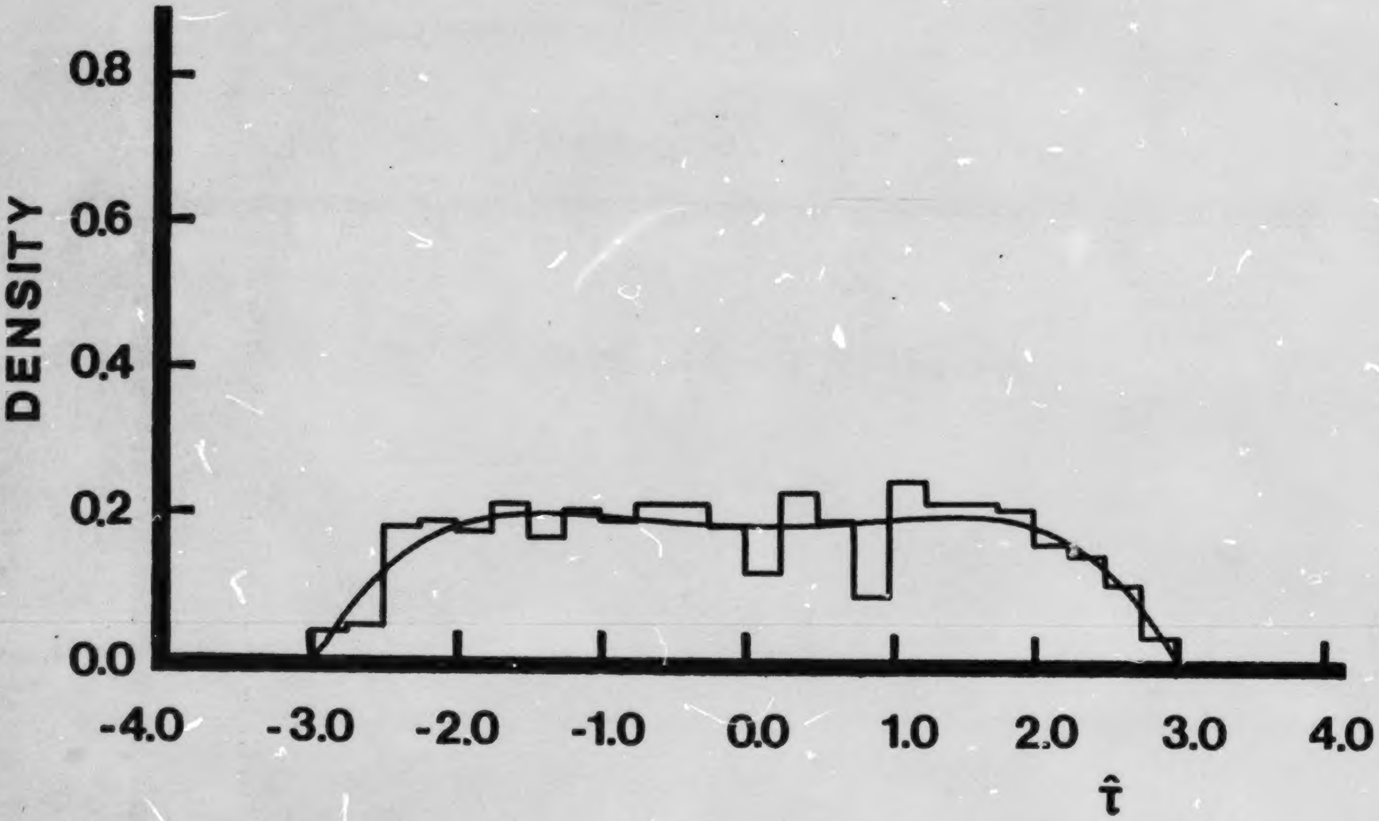


FIGURE 2-3 (Continued): Subtest 1, $\hat{\tau}$, Polynomial of Degree 4 .

density function, $g(\hat{\tau})$, are also presented by solid curves, respectively. Hereafter, we shall call the case in which the polynomial of degree 3 is used for $\hat{g}(\hat{\tau})$ Degree 3 Case, and the one in which that of degree 4 is used Degree 4 Case, as we did in previous studies. These two polynomials are:

$$\hat{g}(\hat{\tau}) = 0.21204 - 0.00092\hat{\tau} - 0.01463\hat{\tau}^2 + 0.00016\hat{\tau}^3 \tag{2.12}$$

in Degree 3 Case, and

$$\hat{g}(\hat{\tau}) = 0.18470 - 0.00198\hat{\tau} + 0.01688\hat{\tau}^2 + 0.00044\hat{\tau}^3 - 0.00424\hat{\tau}^4 \quad , \tag{2.13}$$

in Degree 4 Case, respectively.

The relative frequency distribution of the five hundred $\hat{\tau}$'s obtained upon Subtest 2 is shown in the form of histograms in each graph of Figure 2-4, and, in the same figure, also presented are the two polynomials of degrees 3 and 4, which were obtained by the method of moments, and are used as estimates of the density function, $g(\hat{\tau})$. They are:

$$\hat{g}(\hat{\tau}) = 0.27318 - 0.00149\hat{\tau} - 0.03584\hat{\tau}^2 + 0.00102\hat{\tau}^3 \tag{2.14}$$

for Degree 3 Case, and

$$\hat{g}(\hat{\tau}) = 0.29301 - 0.00185\hat{\tau} - 0.05903\hat{\tau}^2 + 0.00112\hat{\tau}^3 + 0.00317\hat{\tau}^4 \quad , \tag{2.15}$$

for Degree 4 Case, respectively.

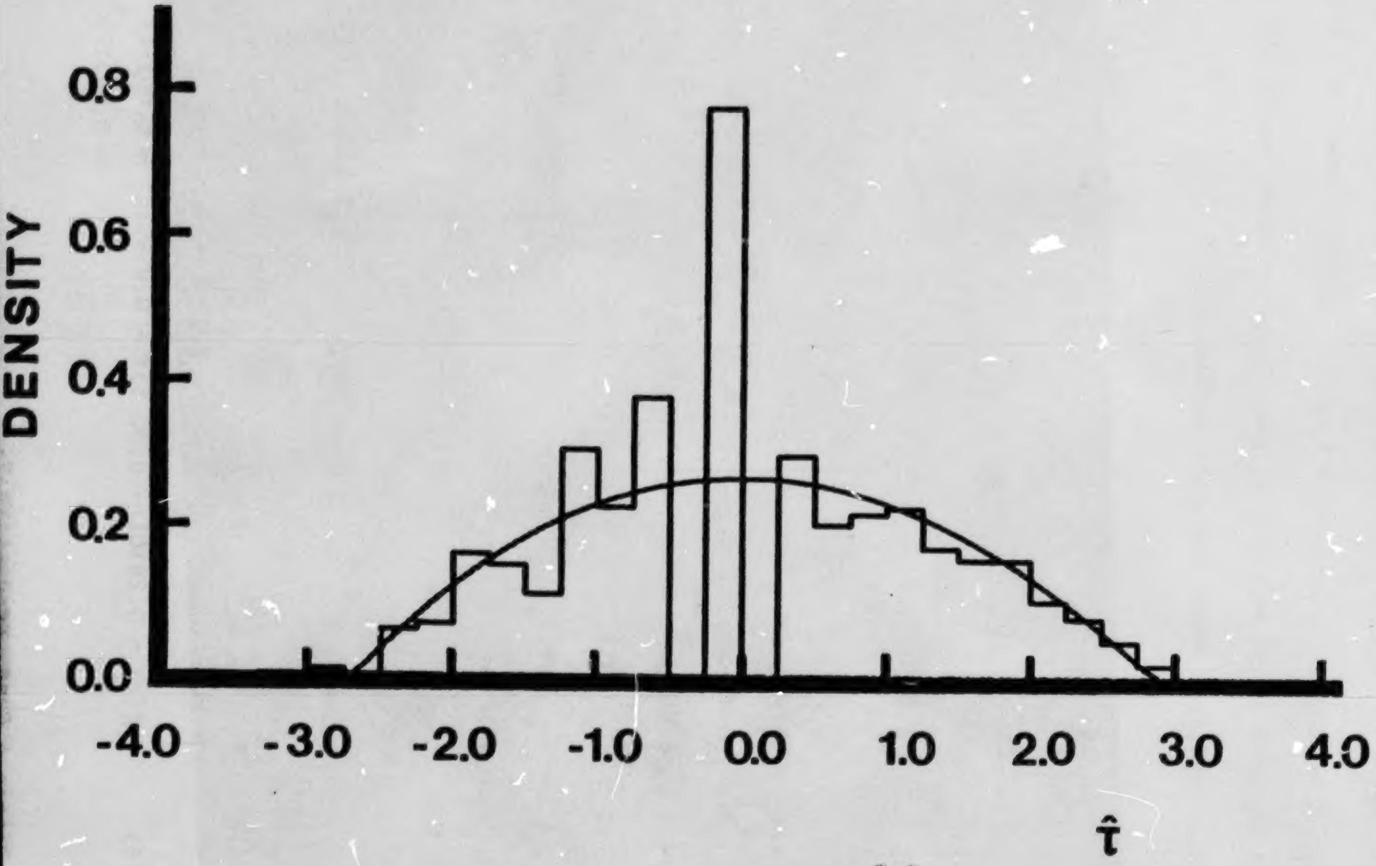


FIGURE 2-4

Relative Frequency Distribution of $\hat{\tau}$, Which Was Obtained for the Five Hundred Hypothetical Examinees on Subtest 2, with 0.25 as the Subinterval Width, Together with the Polynomial of Degree 3 Obtained by the Method of Moments to Approximate the Density Function of $\hat{\tau}$.

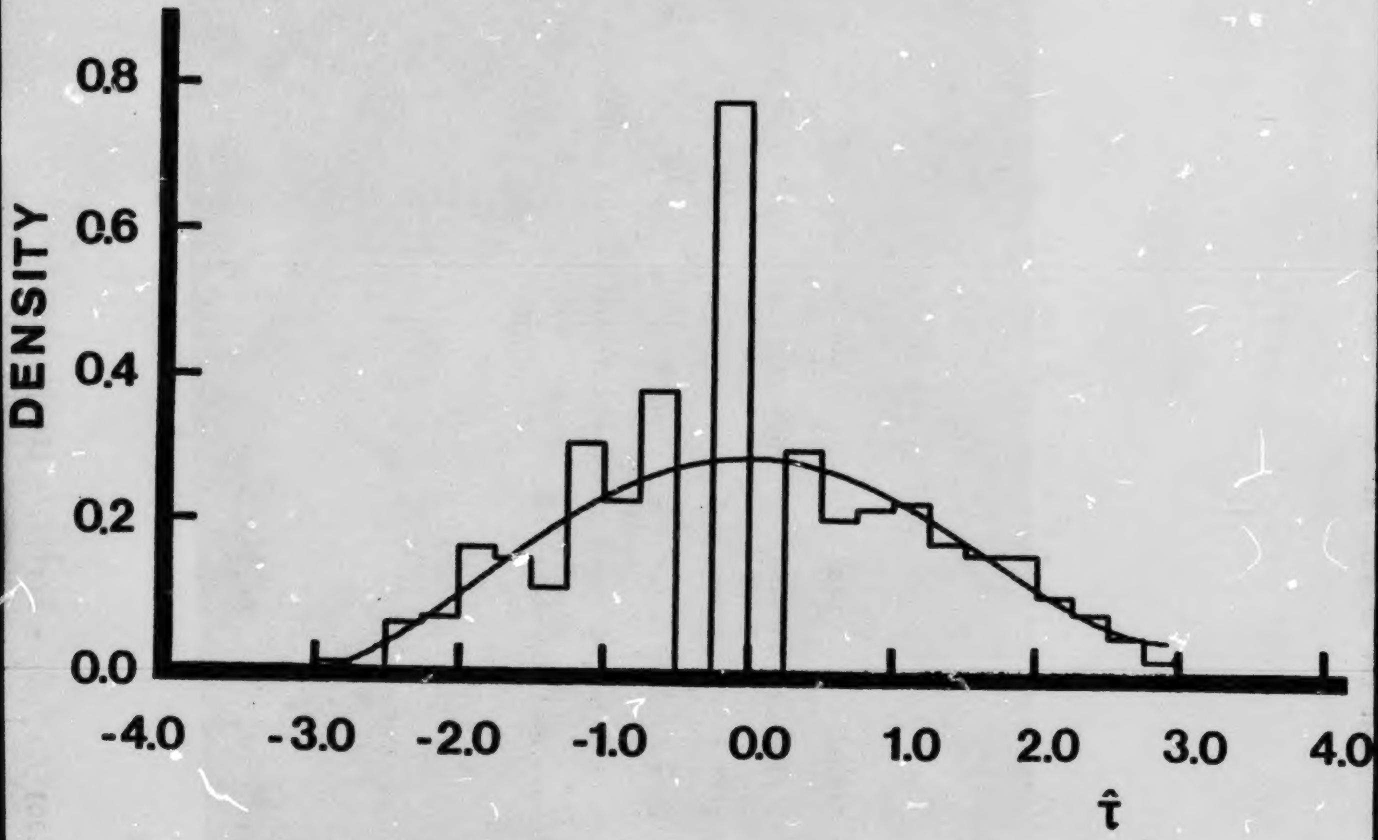


FIGURE 2-4 (Continued): Subtest 2, $\hat{\tau}$, Polynomial of Degree 4 .

We can write for the first and second conditional moments of τ about the origin, given $\hat{\tau}$, such that

$$(2.16) \qquad E(\tau|\hat{\tau}) = \hat{\tau} + C^{-2} \frac{d}{d\hat{\tau}} \log g(\hat{\tau}) \quad ,$$

and

$$(2.17) \qquad E(\tau^2|\hat{\tau}) = \hat{\tau}^2 + 2\hat{\tau}C^{-2} \frac{d}{d\hat{\tau}} \log g(\hat{\tau})$$

$$+ C^{-4}\left[\frac{d^2}{d\hat{\tau}^2} \log g(\hat{\tau}) + \left\{\frac{d}{d\hat{\tau}} \log g(\hat{\tau})\right\}^2 \right] + C^{-2} \quad .$$

The estimates of these two conditional moments, $\hat{E}(\tau|\hat{\tau})$ and $\hat{E}(\tau^2|\hat{\tau})$, are obtained by replacing $g(\hat{\tau})$ in (2.16) and (2.17) by $\hat{g}(\hat{\tau})$, which is given either by (2.14) or by (2.15). Thus the approximated conditional density function, $\hat{\phi}(\tau|\hat{\tau})$, of τ , given $\hat{\tau}$, in the Normal Approach Method of the Conditional P.D.F. Approach can be written as

$$(2.18) \qquad \hat{\phi}(\tau|\hat{\tau}) = (2\pi\hat{\mu}_2)^{-1/2} \exp[-(\tau-\hat{\mu}_1')^2/(2\hat{\mu}_2)] \quad ,$$

where

$$(2.19) \qquad \hat{\mu}_1' = \hat{E}(\tau|\hat{\tau})$$

and

$$(2.20) \qquad \hat{\mu}_2 = \hat{E}(\tau^2|\hat{\tau}) - [\hat{E}(\tau|\hat{\tau})]^2 \quad .$$

The estimated operating characteristic, $\hat{P}_{x_h}(\theta)$, of the discrete item response, x_h , to a new item, h , in the Simple Sum Procedure of the Conditional P.D.F. Approach is given by

$$(2.21) \qquad \hat{P}_{x_h}(\theta) = \sum_{s \varepsilon x_h} \hat{\phi}(\tau|\hat{\tau}_s)[\sum_{s=1}^{N} \hat{\phi}(\tau|\hat{\tau}_s)]^{-1} \quad , \quad x_h = 0,1,\ldots,m_h \ ,$$

where s denotes an individual examinee, N is the number of examinees, i.e., 500 in the present example, and $\hat{\tau}_s$ is the maximum likelihood estimate of τ for the individual s .

Ten binary items were chosen as new items, as was the case in the previous studies. Each of these ten binary items follows the normal ogive model, whose item discrimination parameter, a_h , and item difficulty parameter, b_h , are shown in Table 2-3. The resulting ten estimated item characteristic functions of Degree 3 Case which is based upon Subtest 1 are presented in Figure 2-5 by dotted curves. In each graph of the same figure, also presented is the theoretical item characteristic function, which is drawn by a solid curve, the frequency ratios of the correct answer to the item, which are shown by a jagged solid curve, and the estimated item characteristic function of Degree 3 Case based upon the original Old Test, which is drawn by a dashed curve. Two arrows in each graph indicate the two endpoints of the interval, in which the ability levels of all the five hundred hypothetical examinees are located.

Figure 2-6 presents the corresponding results of Degree 4 Case, which are also based upon Subtest 1. We notice that they are very

TABLE 2-3

Item Discrimination Parameter, a_h , and Item Difficulty Parameter, b_h , of Each of Ten Binary Items

Item h	a_h	b_h
1	1.5	-2.5
2	1.0	-2.0
3	2.5	-1.5
4	1.0	-1.0
5	1.5	-0.5
6	1.0	0.0
7	2.0	0.5
8	1.0	1.0
9	2.0	1.5
10	1.0	2.0

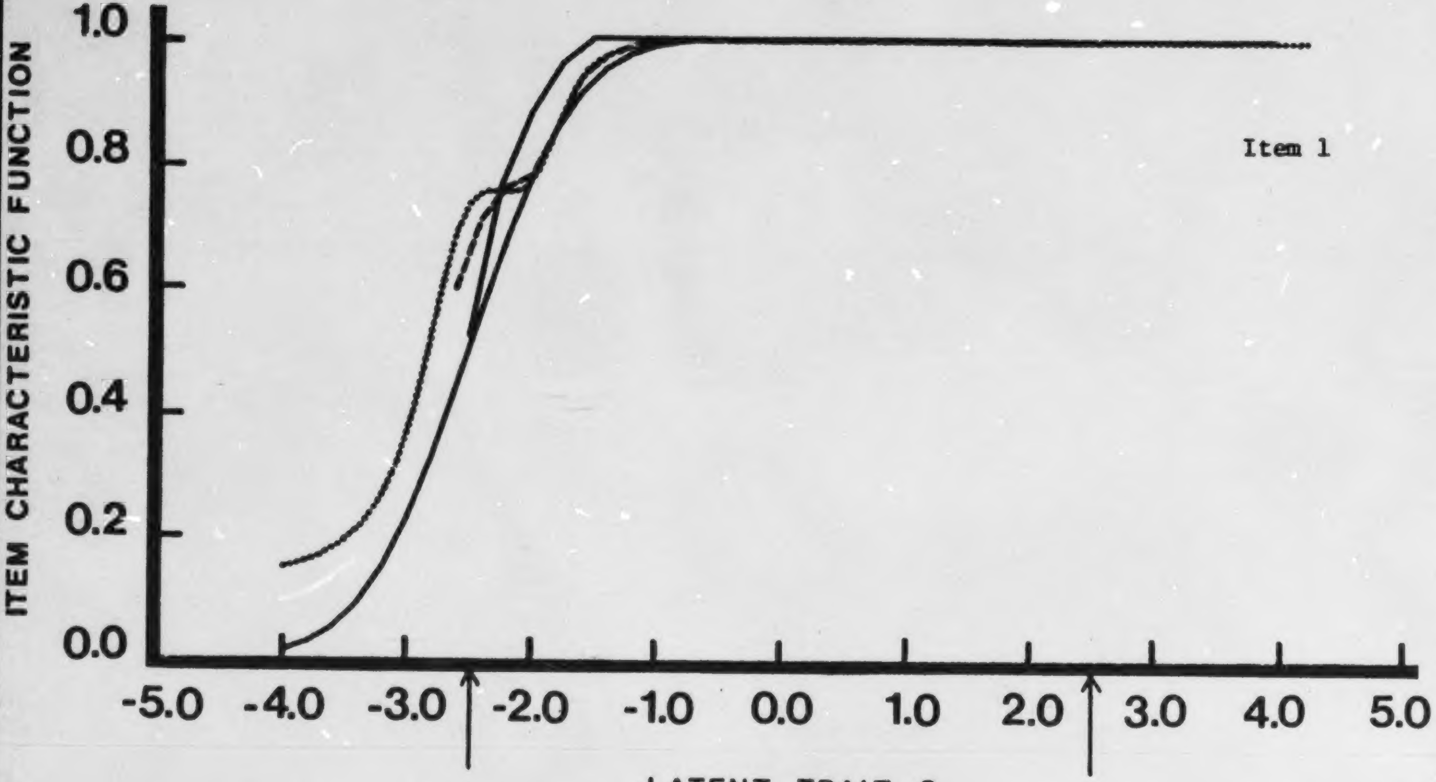


FIGURE 2-5

Estimated Item Characteristic Function Based Upon Subtest 1 (Dotted Curve), in Comparison with the One Based Upon the Total Old Test (Dashed Curve), Obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method. Theoretical Item Characteristic Function (Smooth Solid Curve) and Frequency Ratios of the Correct Answer (Jagged Solid Curve) are Also Presented.
Degree 3 Case

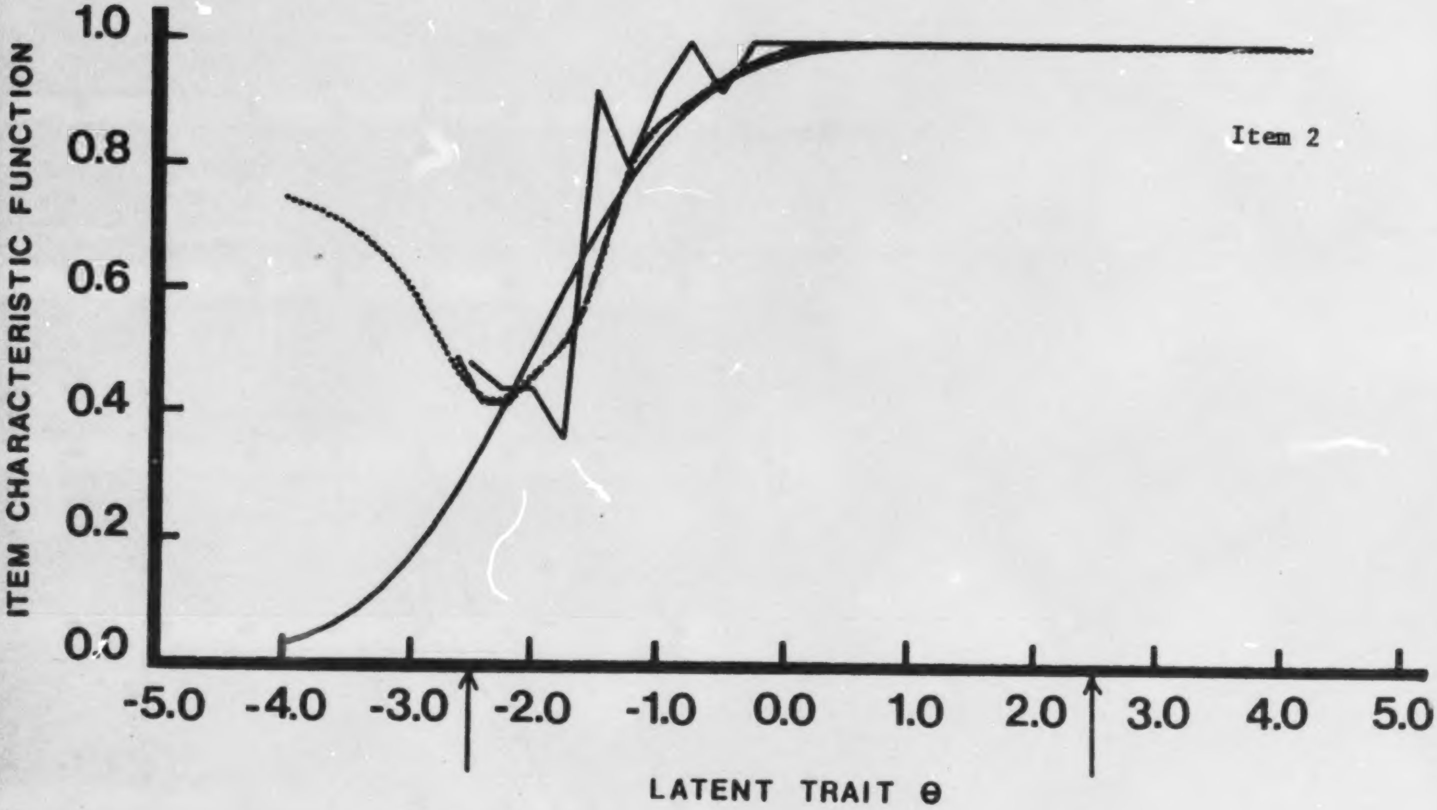


FIGURE 2-5 (Continued): Subtest 1, Degree 3 Case.

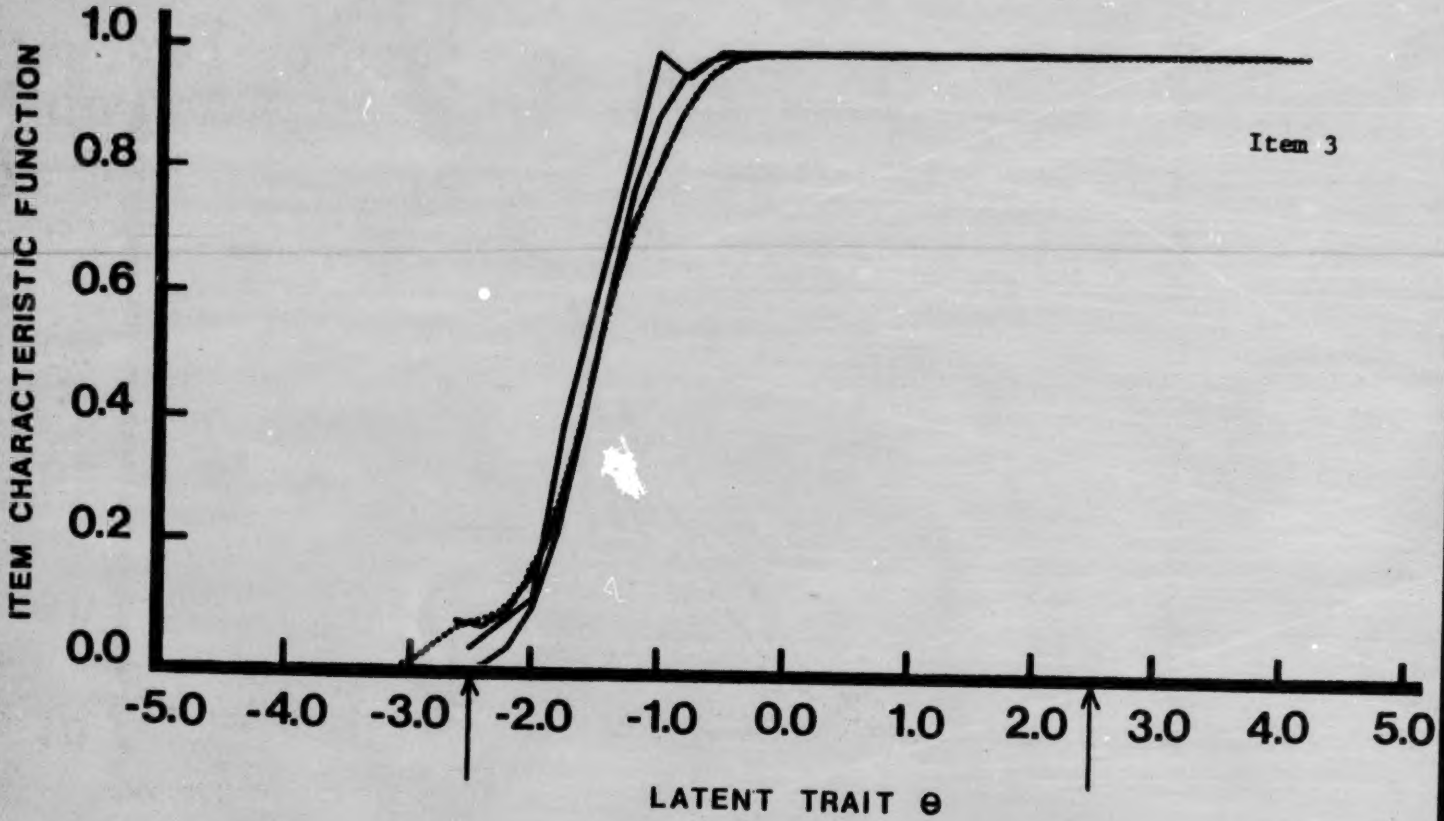


FIGURE 2-5 (Continued): Subtest 1, Degree 3 Case.

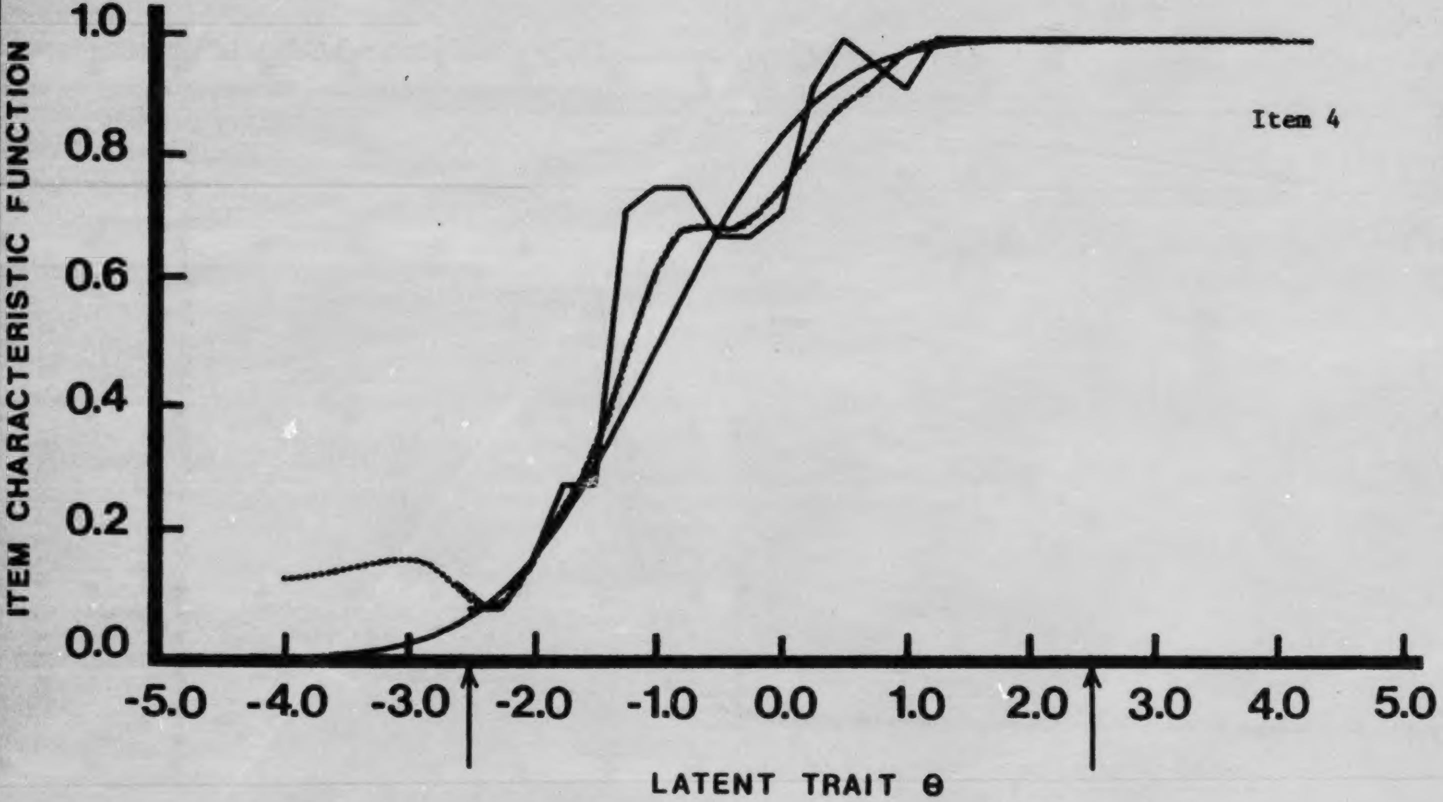


FIGURE 2-5 (Continued): Subtest 1, Degree 3 Case.

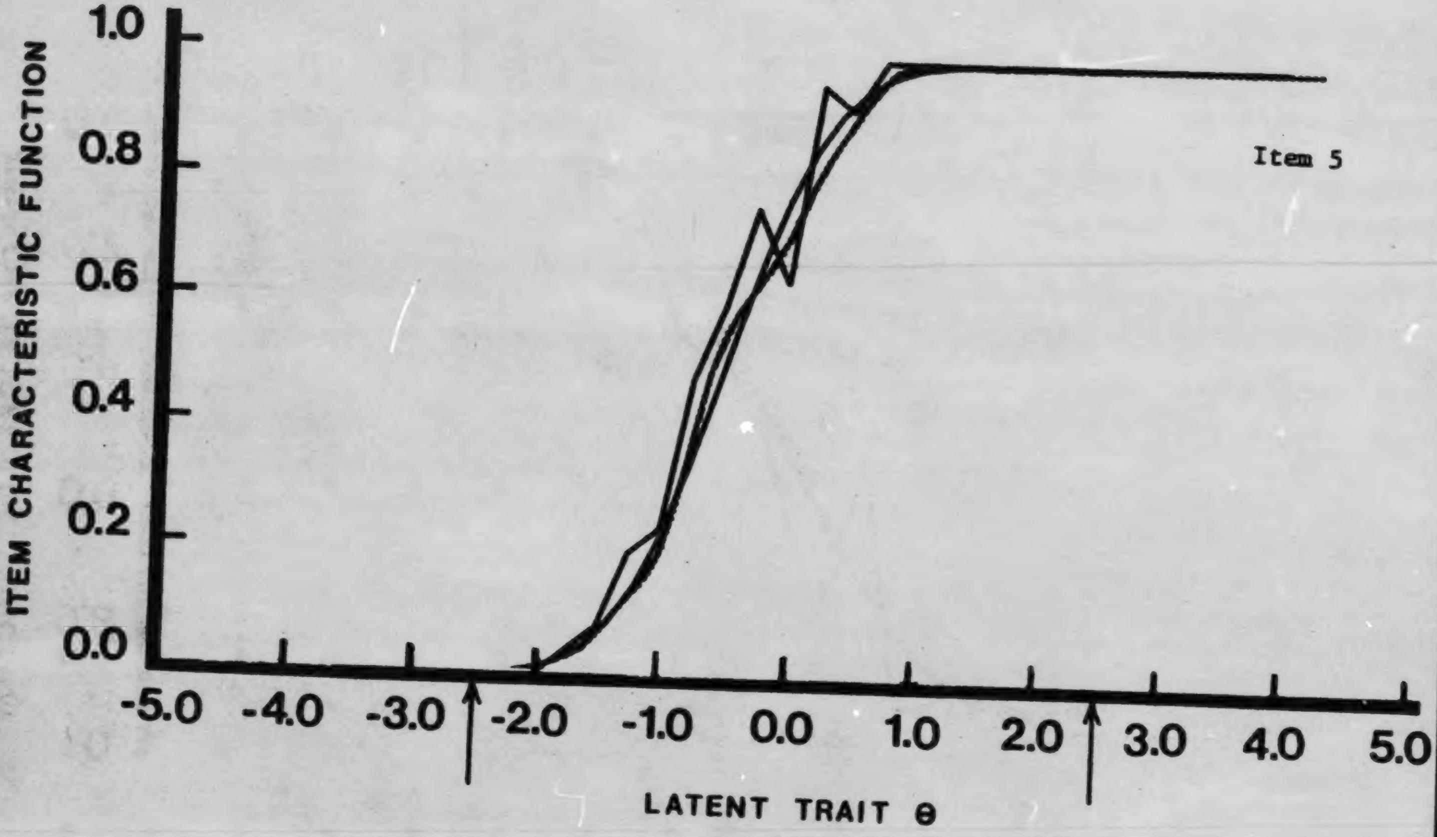


FIGURE 2-5 (Continued): Subtest 1, Degree 3 Case.

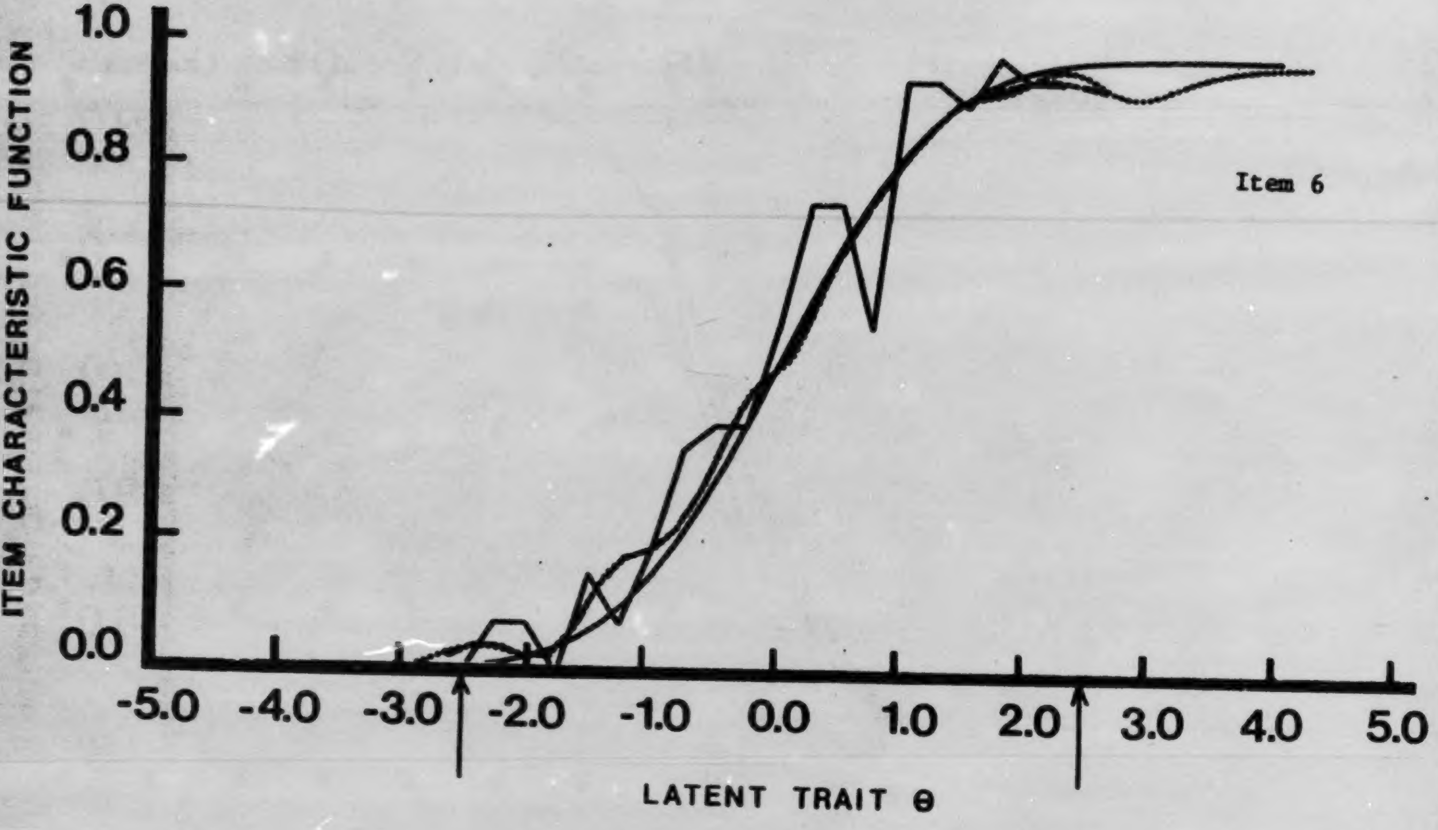


FIGURE 2-5 (Continued): Subtest 1, Degree 3 Case.

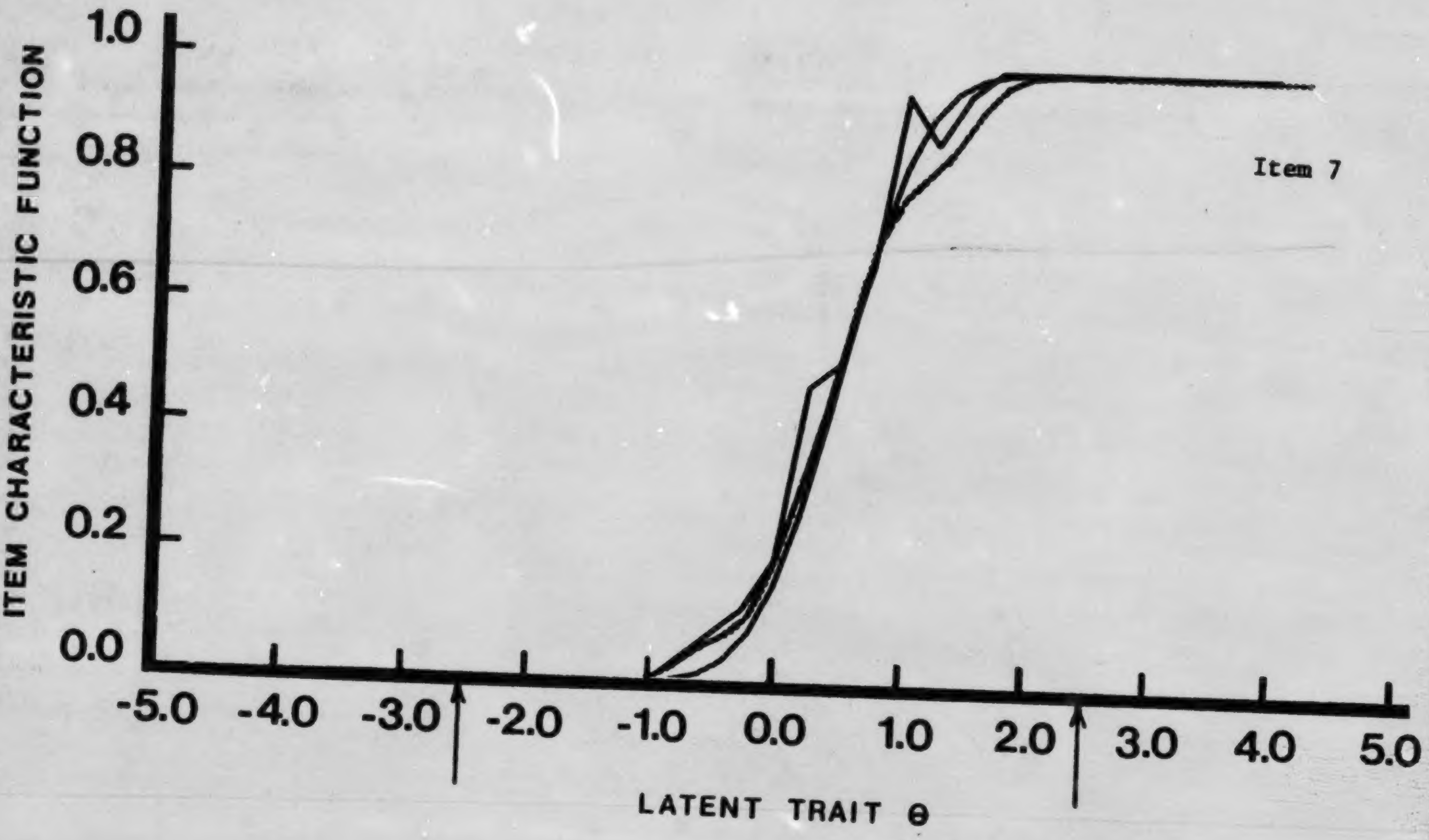


FIGURE 2-5 (Continued): Subtest 1, Degree 3 Case.

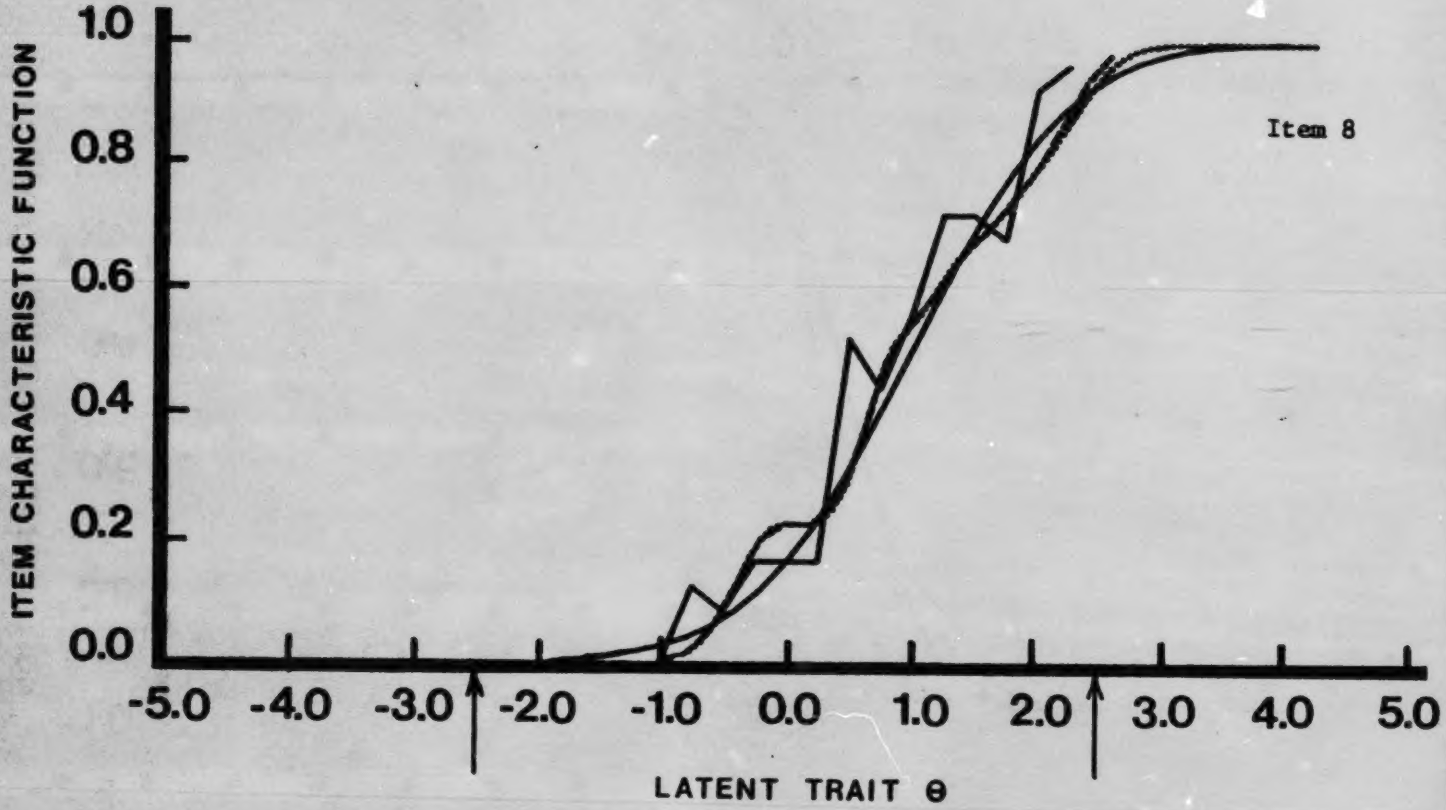


FIGURE 2-5 (Continued): Subtest 1, Degree 3 Case.

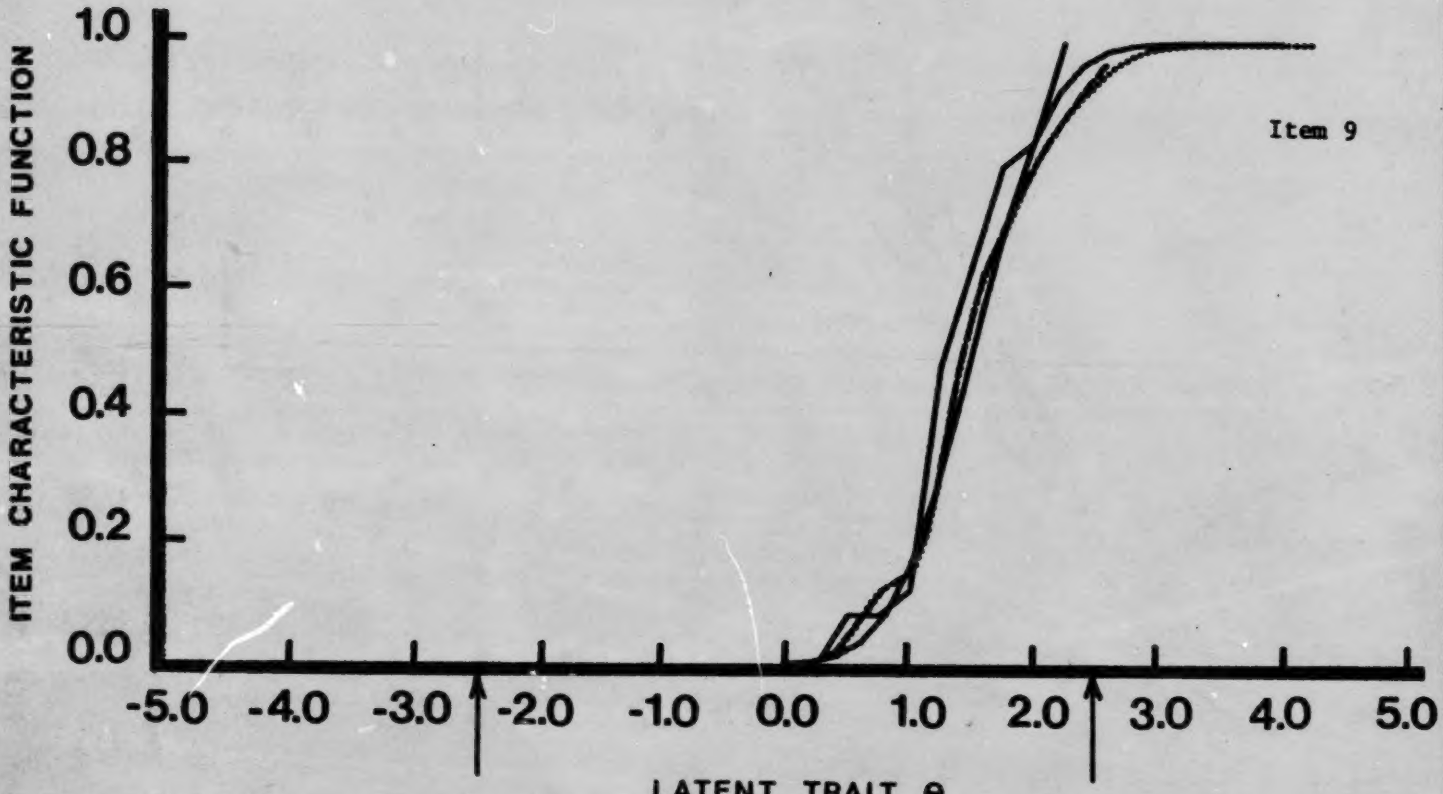


FIGURE 2-5 (Continued): Subtest 1, Degree 3 Case.

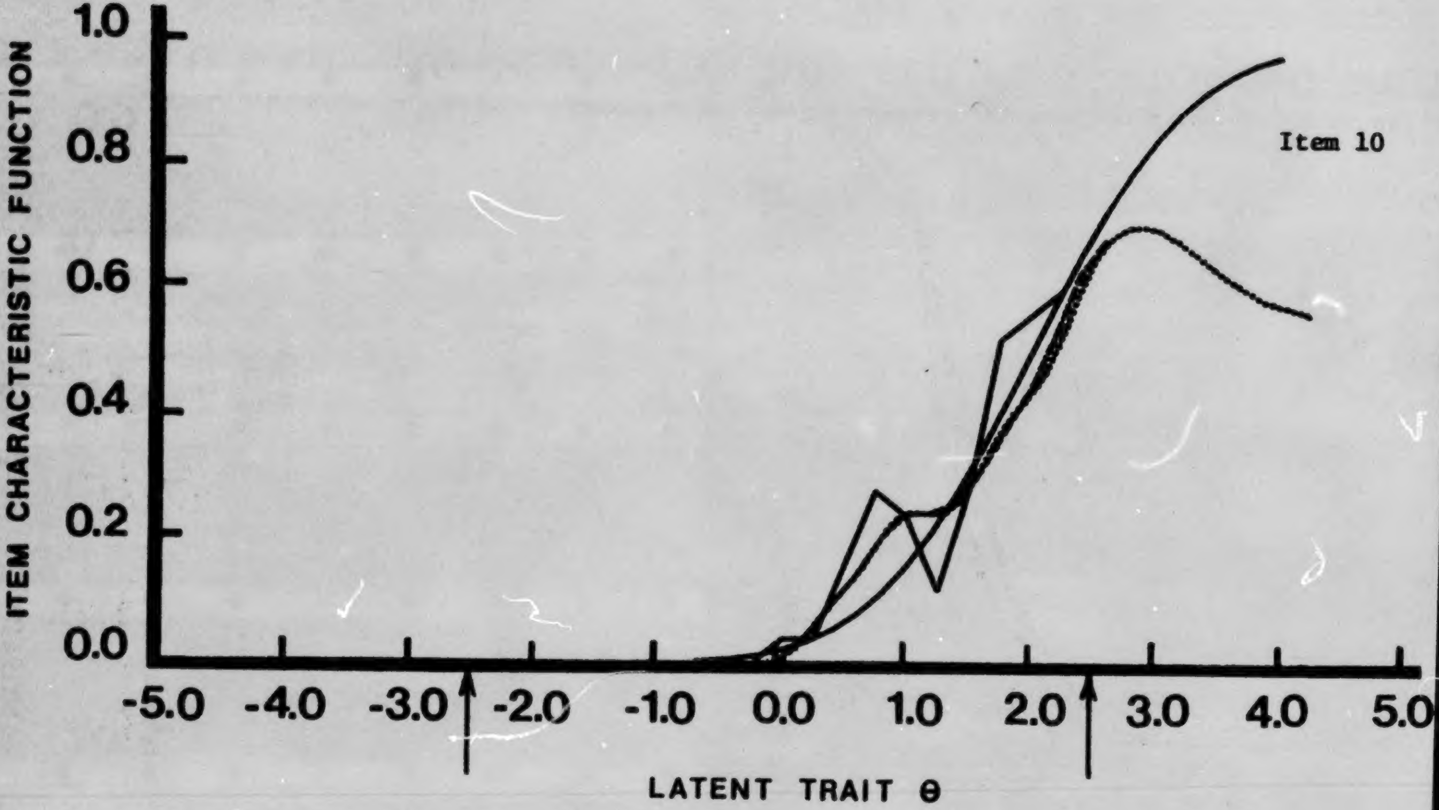


FIGURE 2-5 (Continued): Subtest 1, Degree 3 Case.

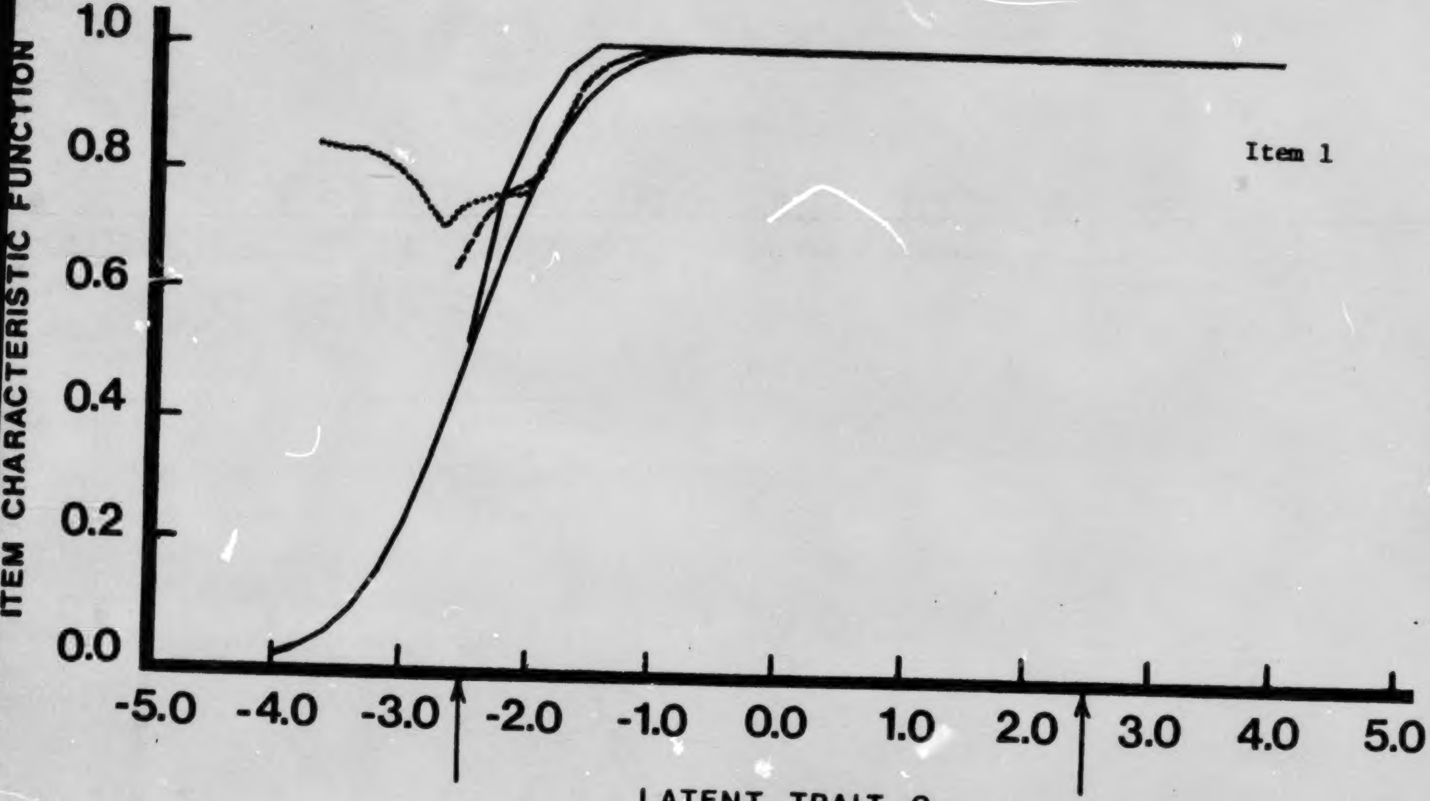


FIGURE 2-6

Estimated Item Characteristic Function Based Upon Subtest 1 (Dotted Curve), in Comparison with the One Based Upon the Total Old Test (Dashed Curve), Obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method. Theoretical Item Characteristic Function (Smooth Solid Curve) and Frequency Ratios of the Correct Answer (Jagged Solid Curve) are Also Presented.
Degree 4 Case

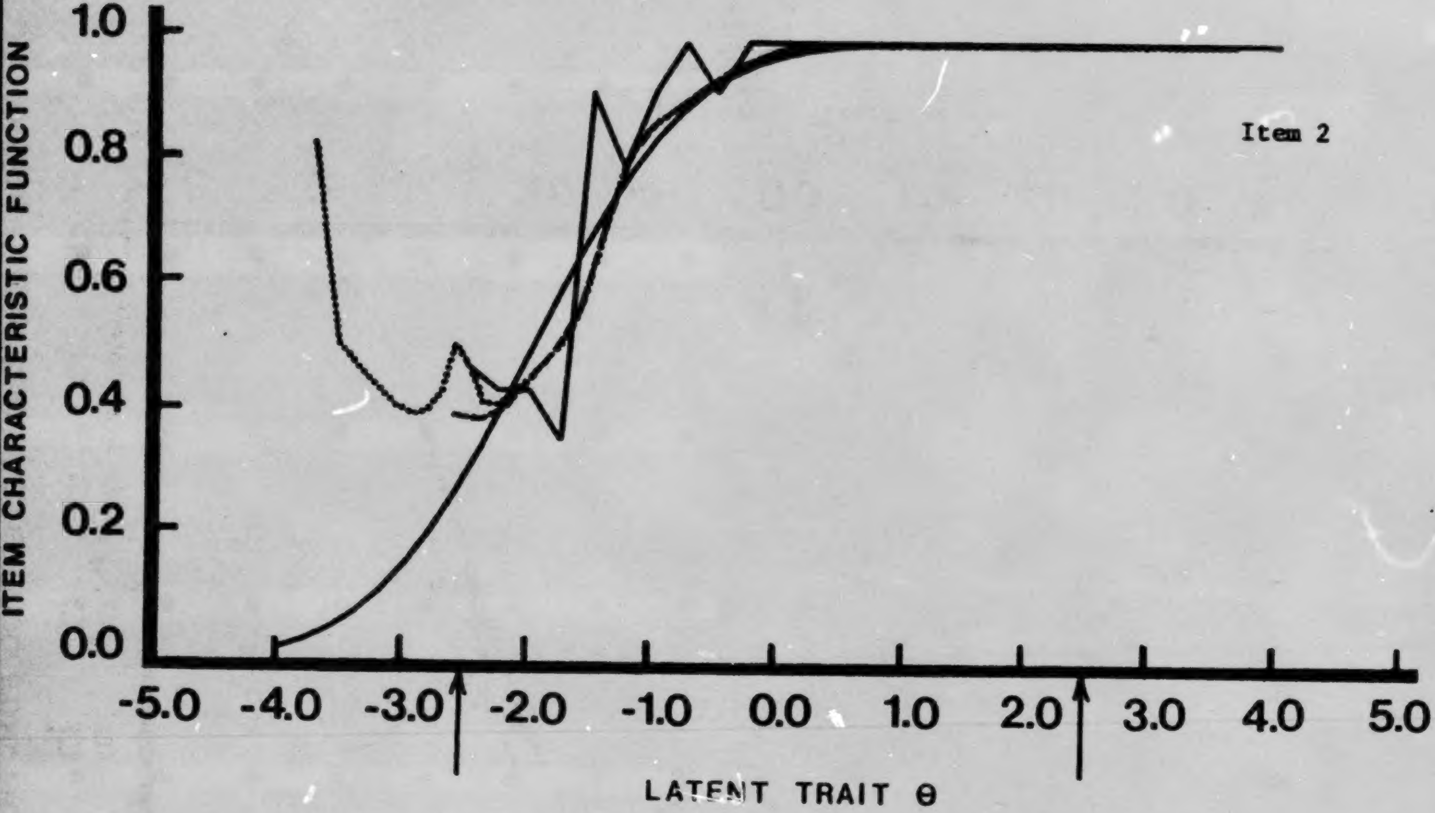


FIGURE 2-6 (Continued): Subtest 1, Degree 4 Case.

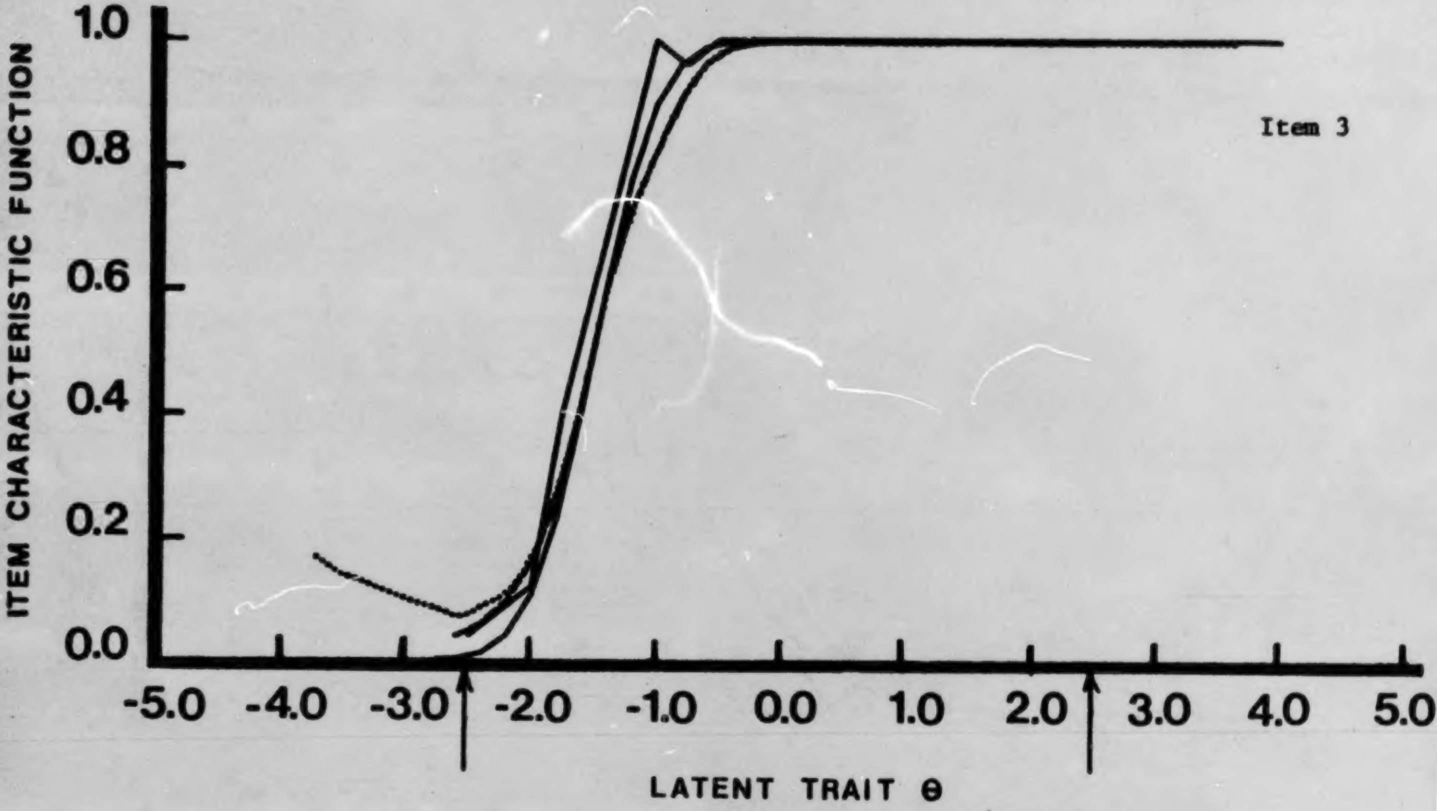


FIGURE 2-6 (Continued): Subtest 1, Degree 4 Case.

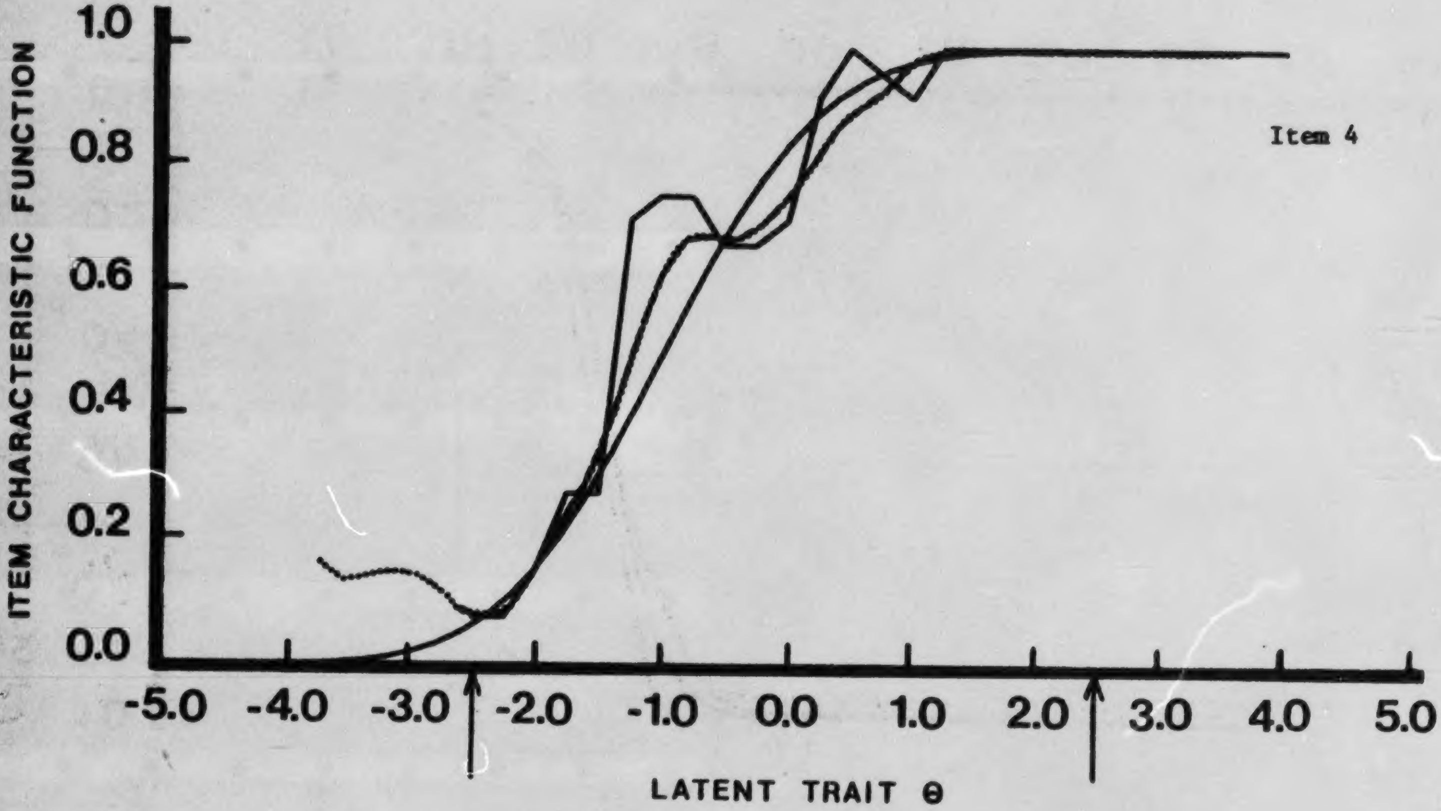


FIGURE 2-6 (Continued): Subtest 1, Degree 4 Case.

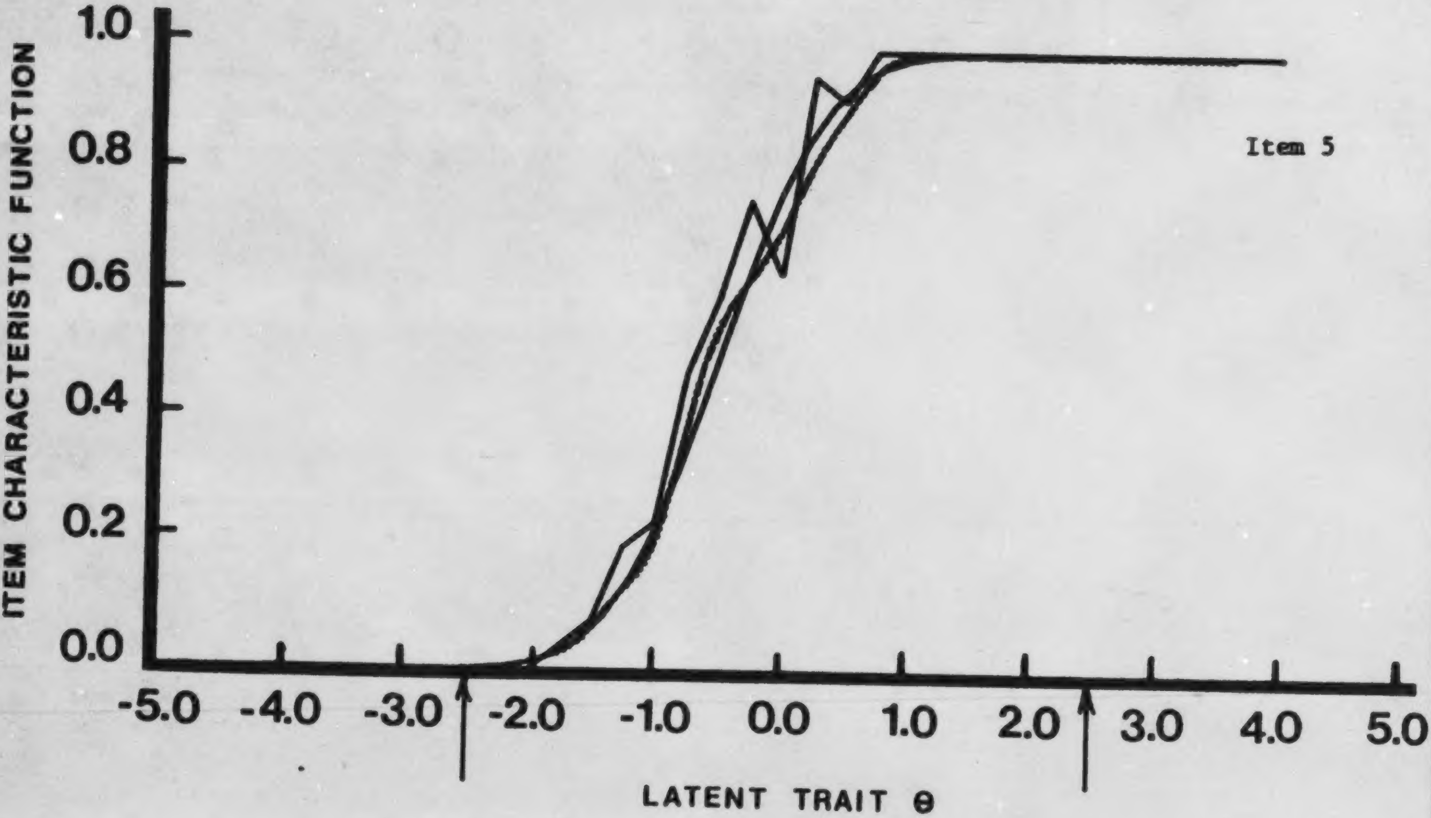


FIGURE 2-6 (Continued): Subtest 1, Degree 4 Case.

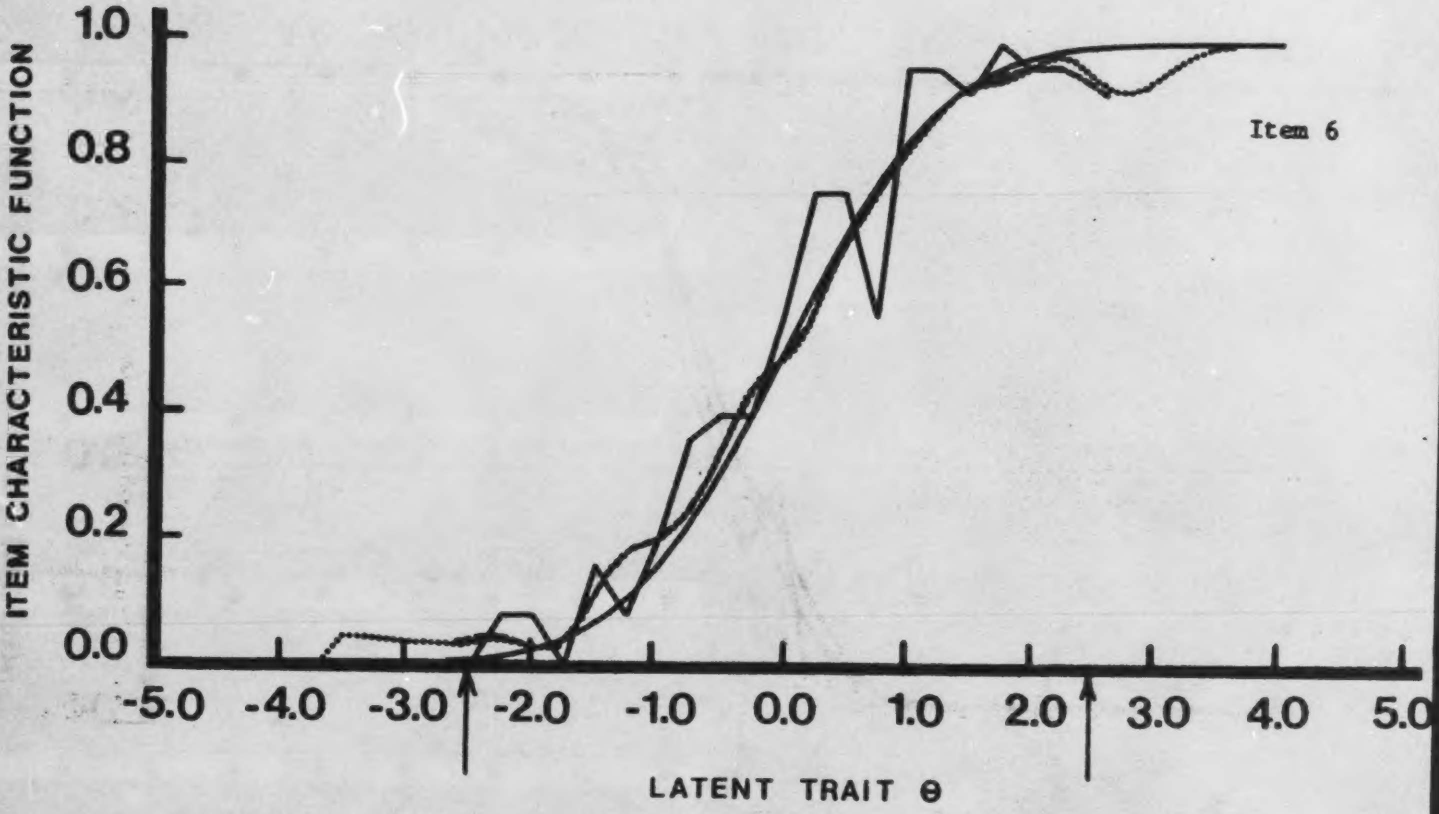


FIGURE 2-6 (Continued): Subtest 1, Degree 4 Case.

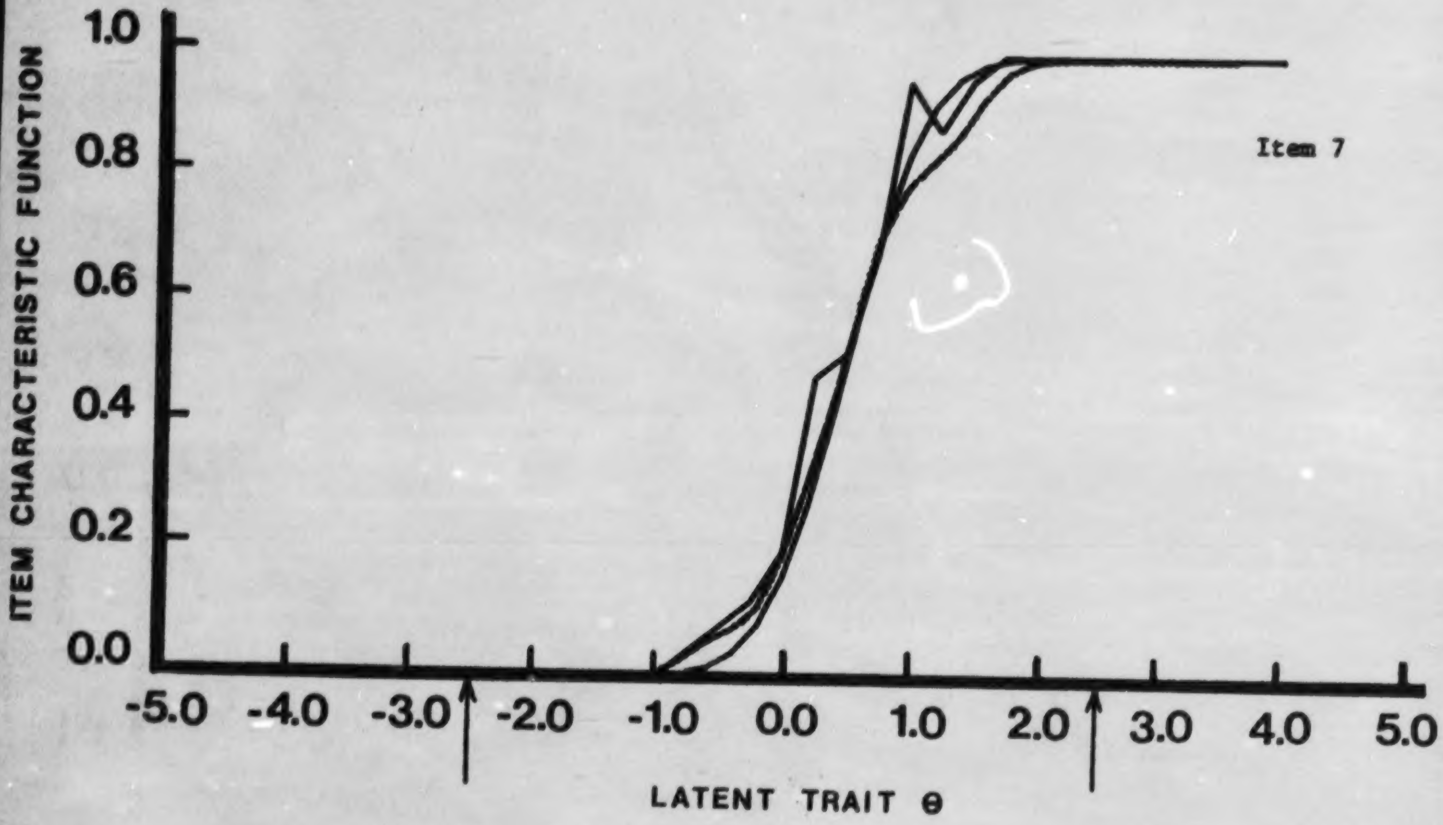


FIGURE 2-6 (Continued): Subtest 1, Degree 4 Case.

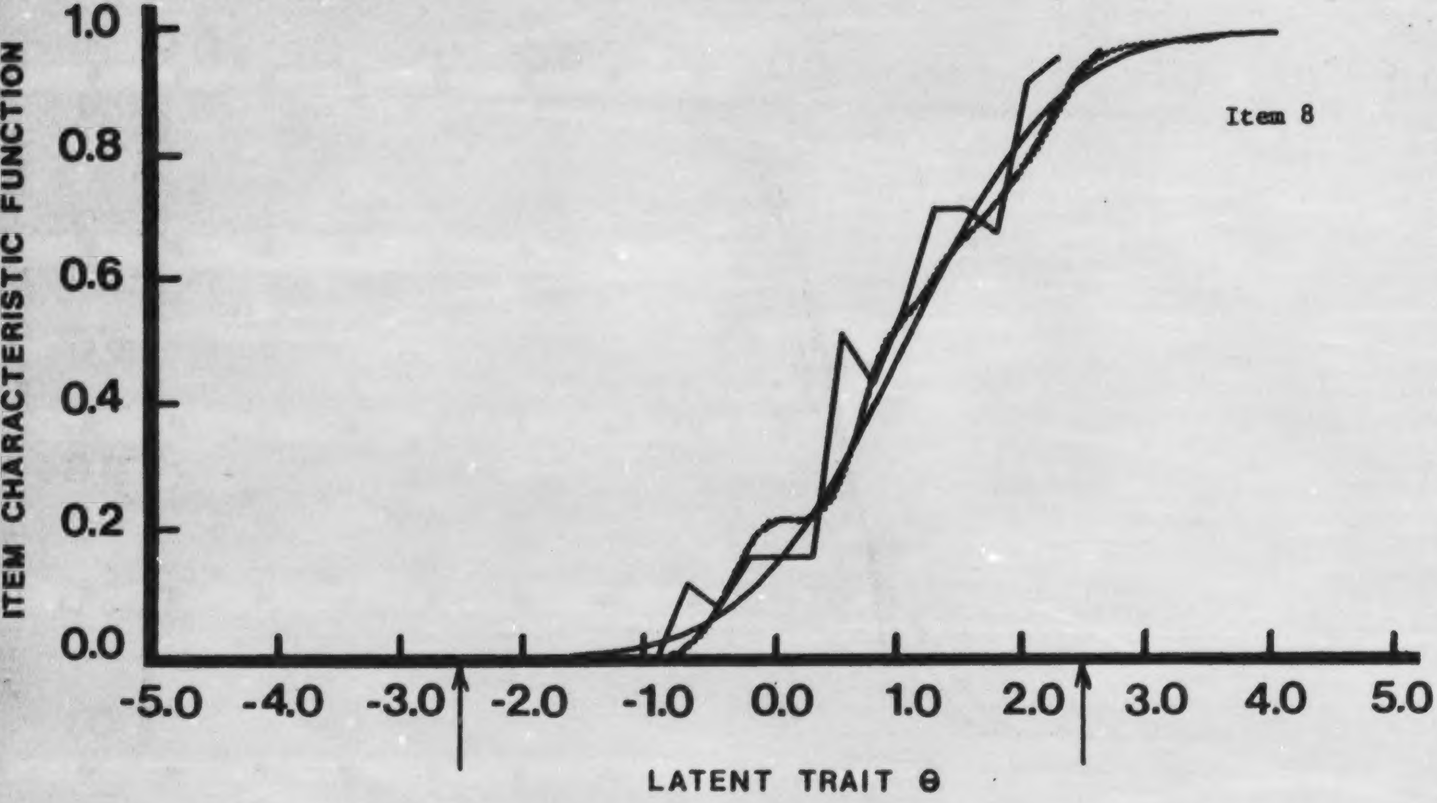


FIGURE 2-6 (Continued): Subtest 1, Degree 4 Case.

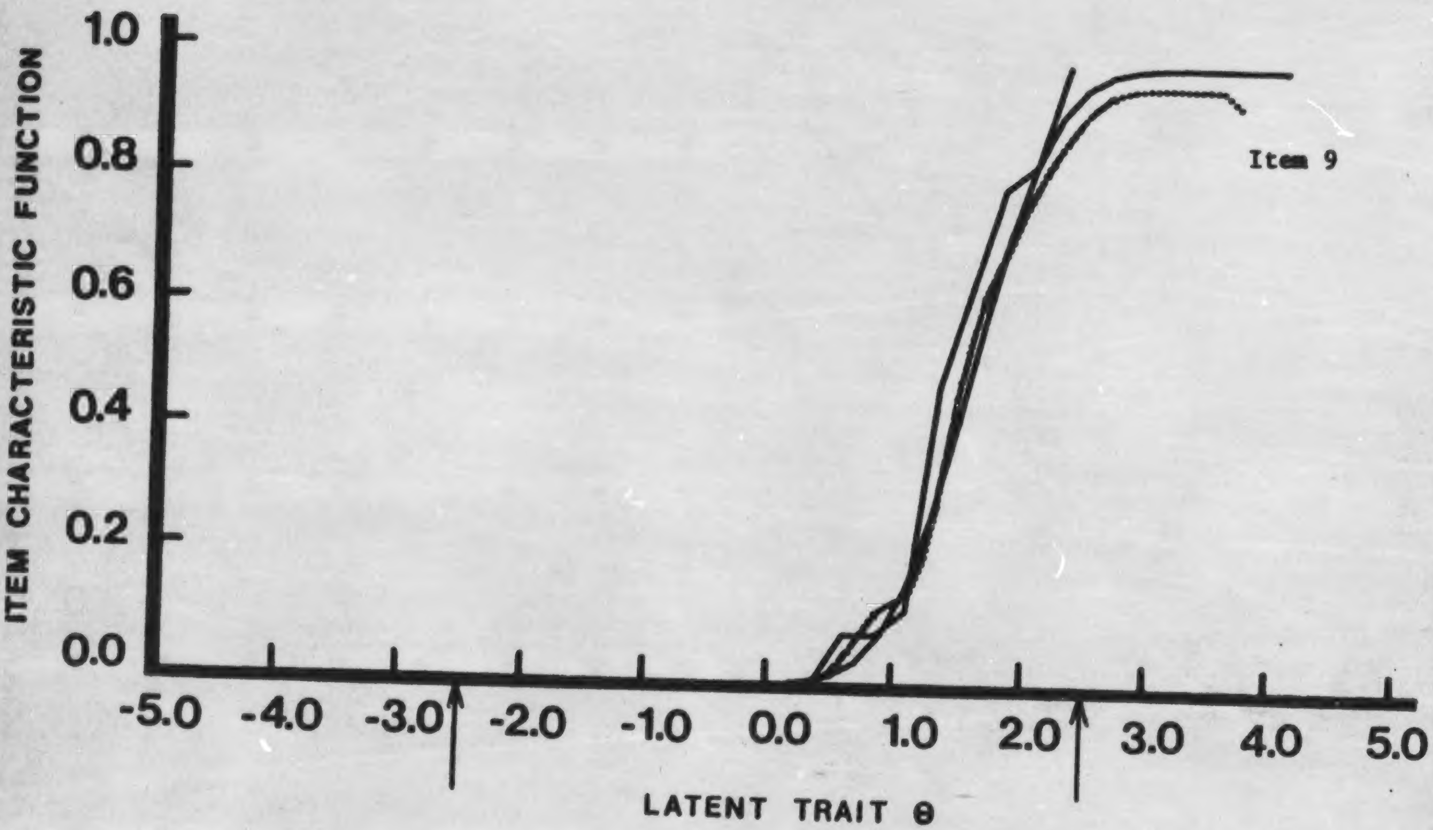


FIGURE 2-6 (Continued): Subtest 1, Degree 4 Case.

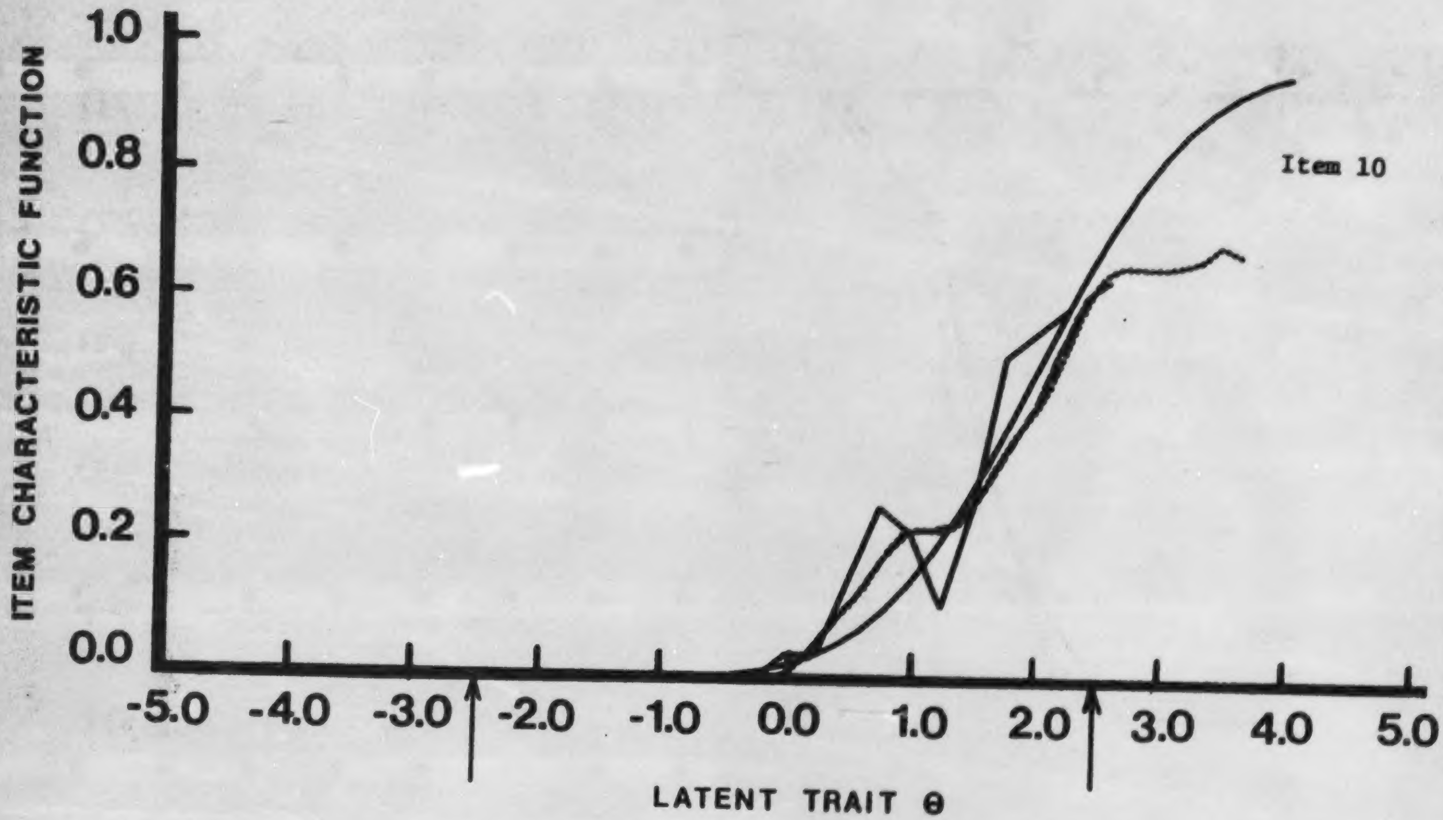


FIGURE 2-6 (Continued): Subtest 1, Degree 4 Case.

similar to those of Degree 3 Case, as was the case with the results based upon the original Old Test.

From these results, we can say that, except for some small discrepancies near the endpoint of the interval, (-2.5, 2.5) , *the estimated item characteristic functions based upon Subtest 1 are practically identical with those obtained upon the original Old Test*; a great success! It is recalled that the latter proved to be practically identical with the criterion item characteristic functions (Samejima, 1978b), which indicates that the present method based upon Subtest 1 has provided us with the best possible results Simple Sum Procedure of the Conditional P.D.F. Approach can produce. It should be noted that the frequency ratios of the correct answer shown in Figures 2-5 and 2-6 are the best we can get without using any sophisticated methods such as ours, and they are not even observable with actual data. Considering this fact, we can say that our results are excellent approximations to the theoretical item characteristic functions.

Implications of this success are promising, in view of the fact that *there are only twenty-five items in Subtest 1*, in comparison with thirty-five items in the original Old Test, and the test information function of Subtest 1 is not constant.

The corresponding results of Degree 3 and 4 Cases which are based upon Subtest 2 are presented in Figures 2-7 and 2-8, respectively. We can see that, unlike the results obtained upon Subtest 1, in these two cases, the estimated item characteristic functions are somewhat different from those obtained upon the original Old Test, especially for items 4, 5, 6 and 7. We notice, however, that, in spite of this fact, all the estimated item characteristic functions still provide us with good approximations to the theoretical item characteristic functions. This is especially clear when we compare these results with the corresponding frequency ratios of the correct answers. Comparison of the two figures indicates, again, that the results of Degree 3 and 4 Cases are practically identical.

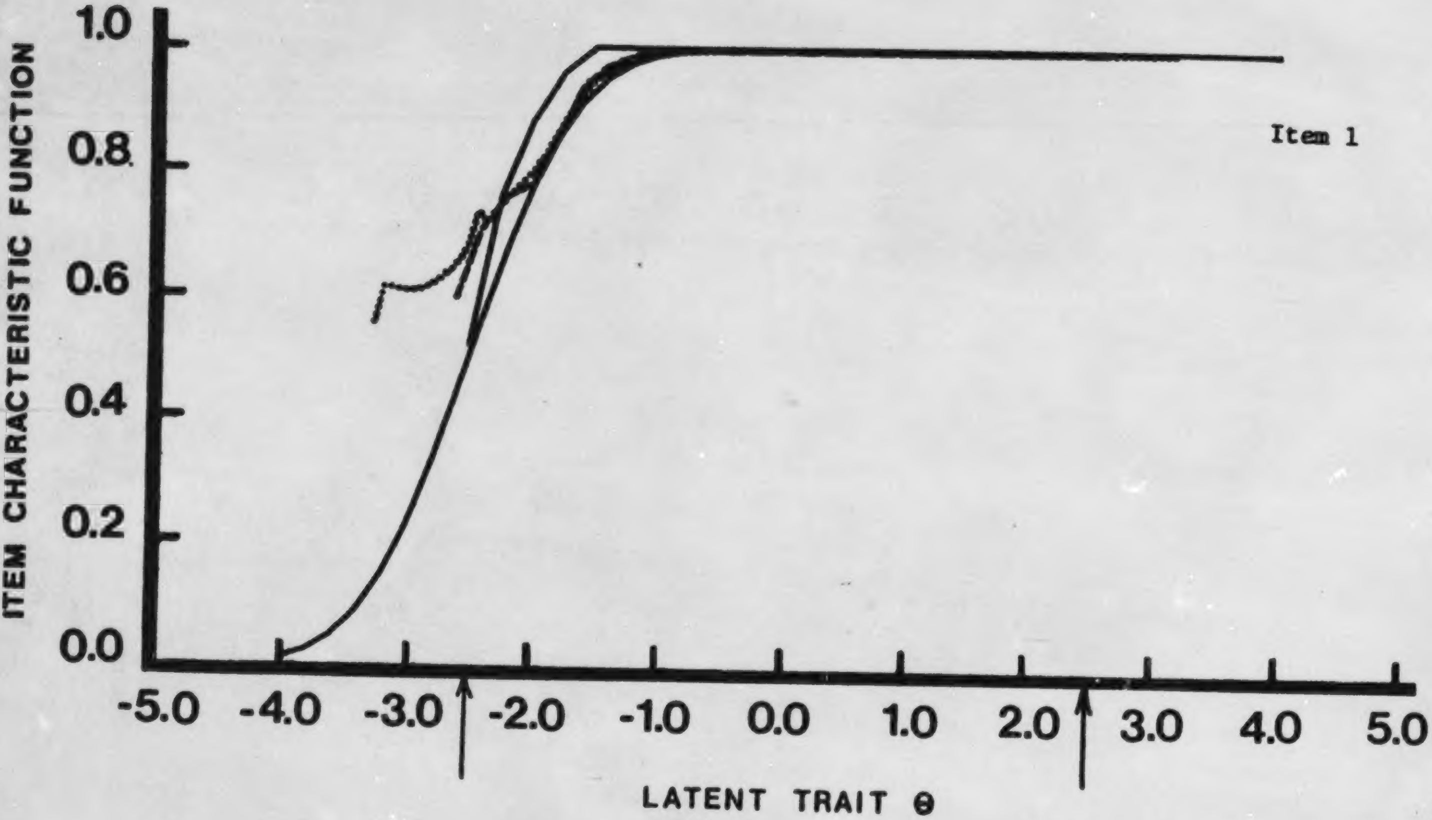


FIGURE 2-7

Estimated Item Characteristic Function Based Upon Subtest 2 (Dotted Curve), in Comparison with the One Based Upon the Total Old Test (Dashed Curve), Obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method. Theoretical Item Characteristic Function (Smooth Solid Curve) and Frequency Ratios of the Correct Answer (Jagged Solid Curve) are Also Presented.
Degree 3 Case

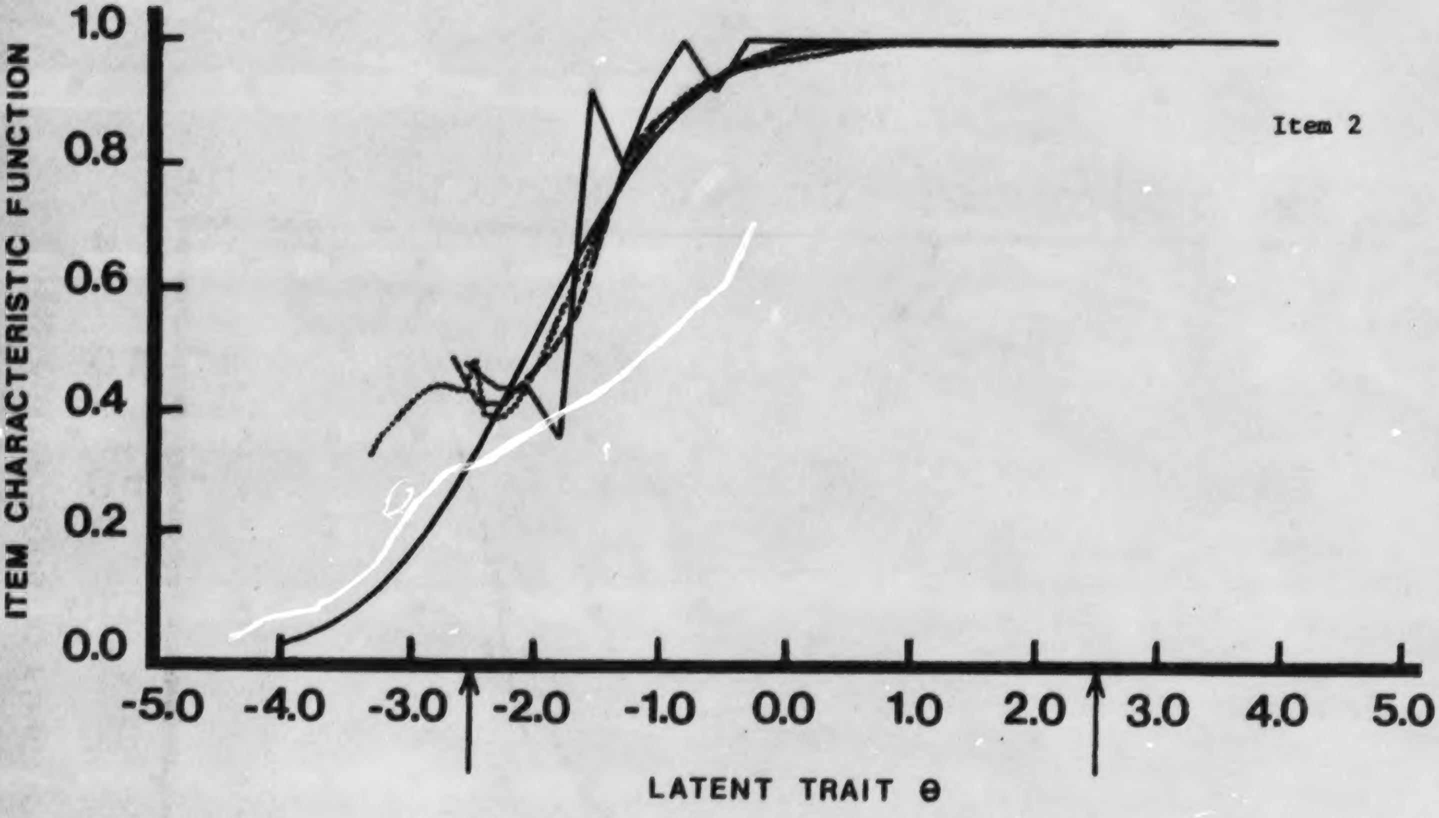


FIGURE 2-7 (Continued): Subtest 2, Degree 3 Case.

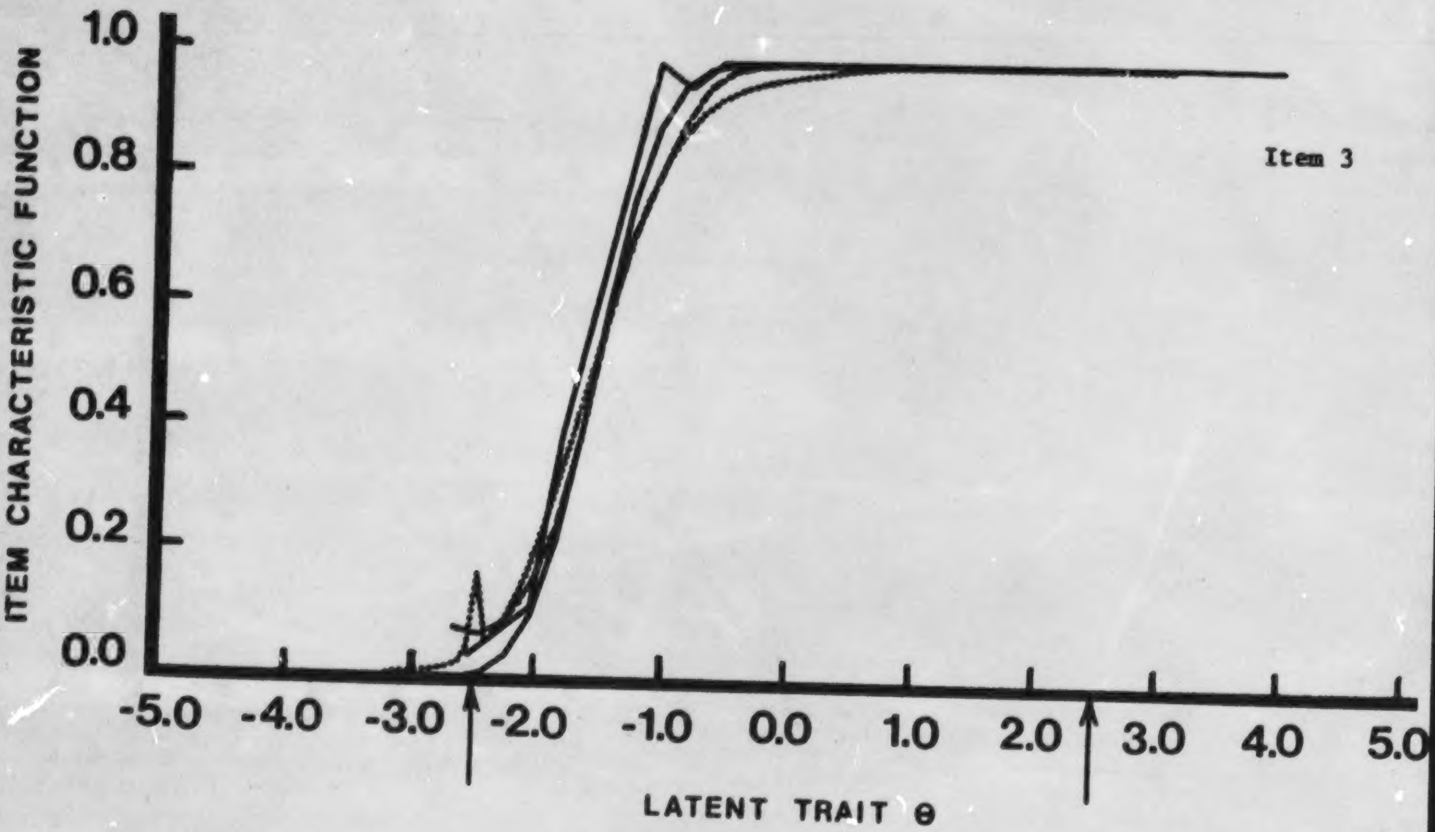


FIGURE 2-7 (Continued): Subtest 2, Degree 3 Case.

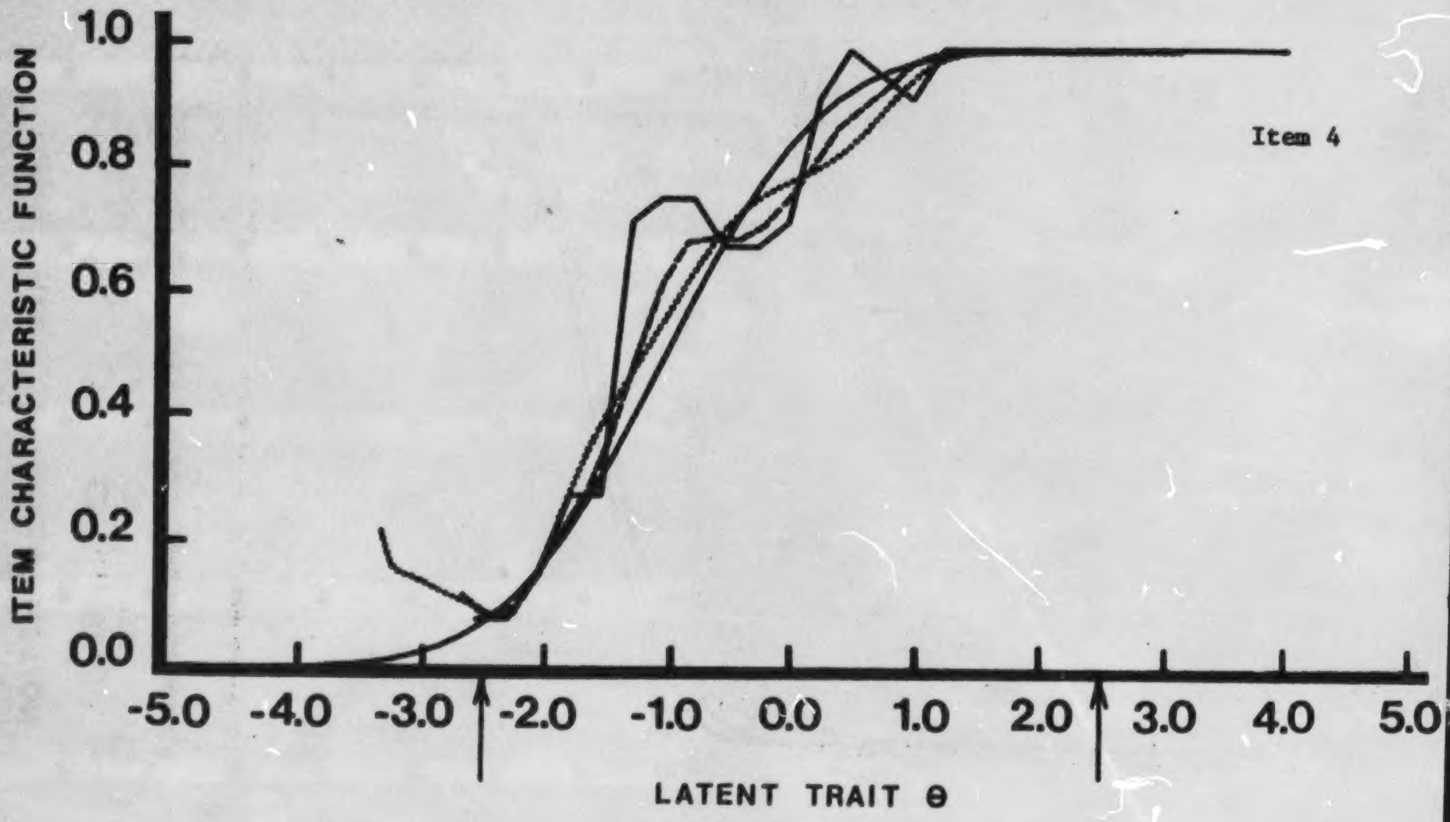


FIGURE 2-7 (Continued): Subtest 2, Degree 3 Case.

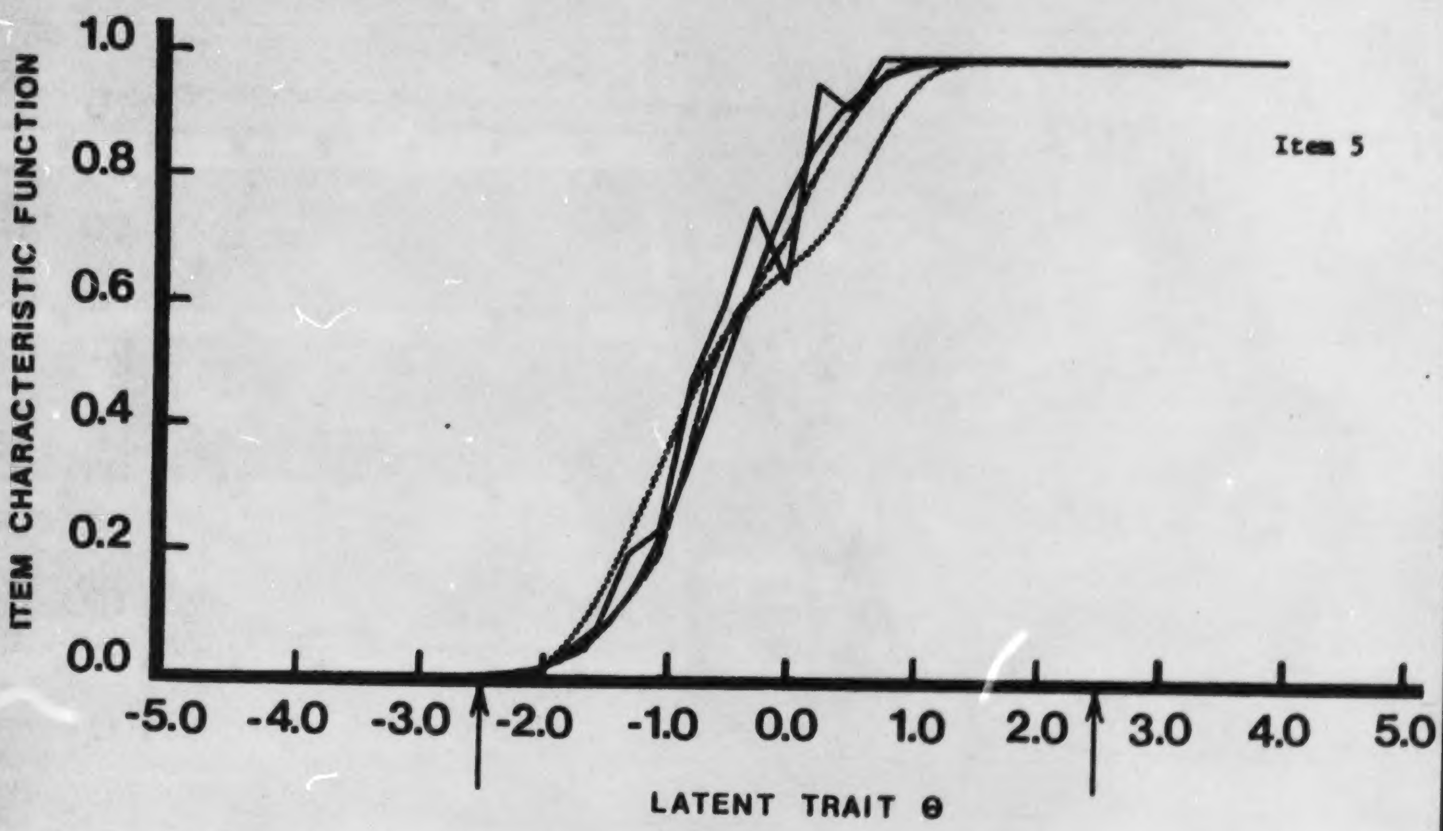


FIGURE 2-7 (Continued): Subtest 2, Degree 3 Case.

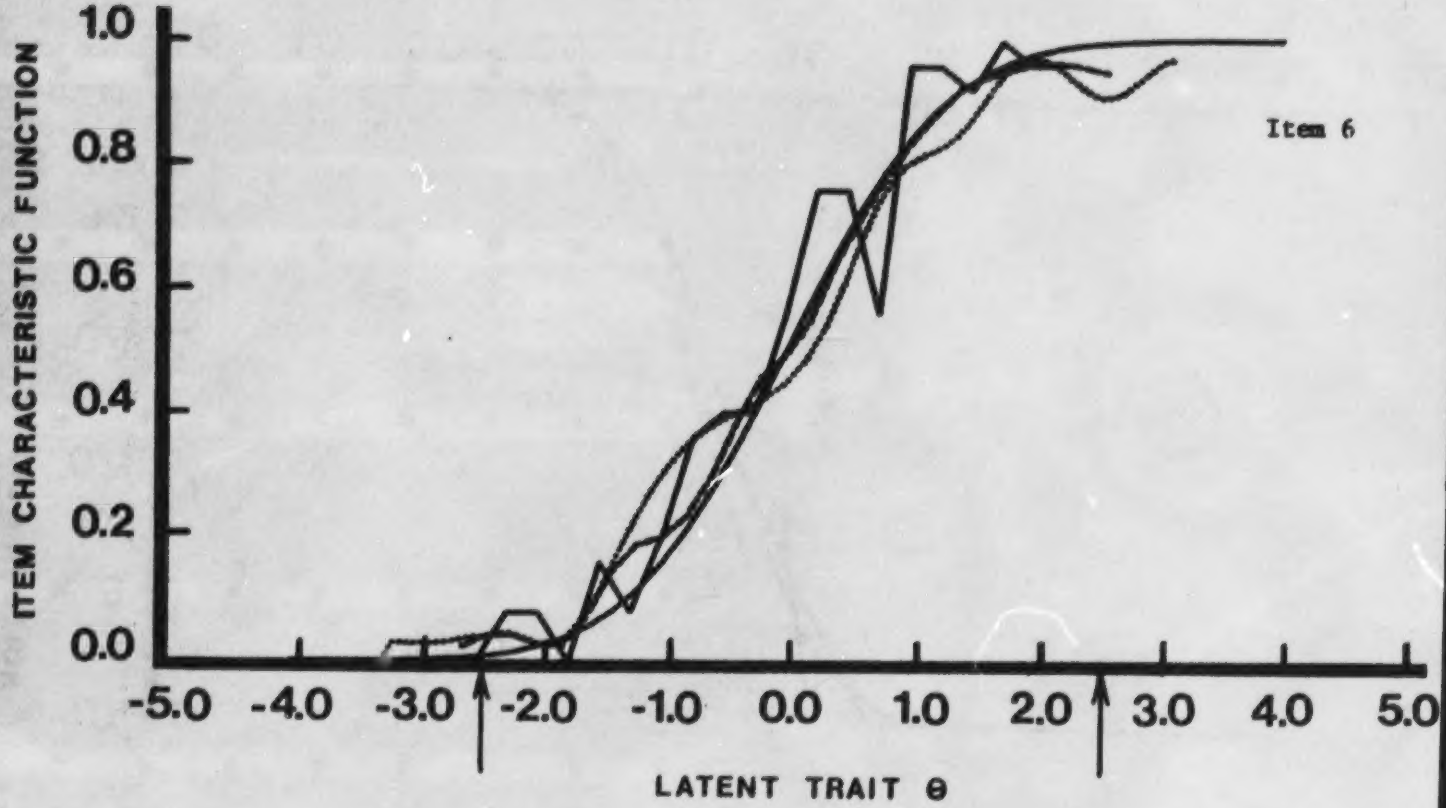


FIGURE 2-7 (Continued): Subtest 2, Degree 3 Case.

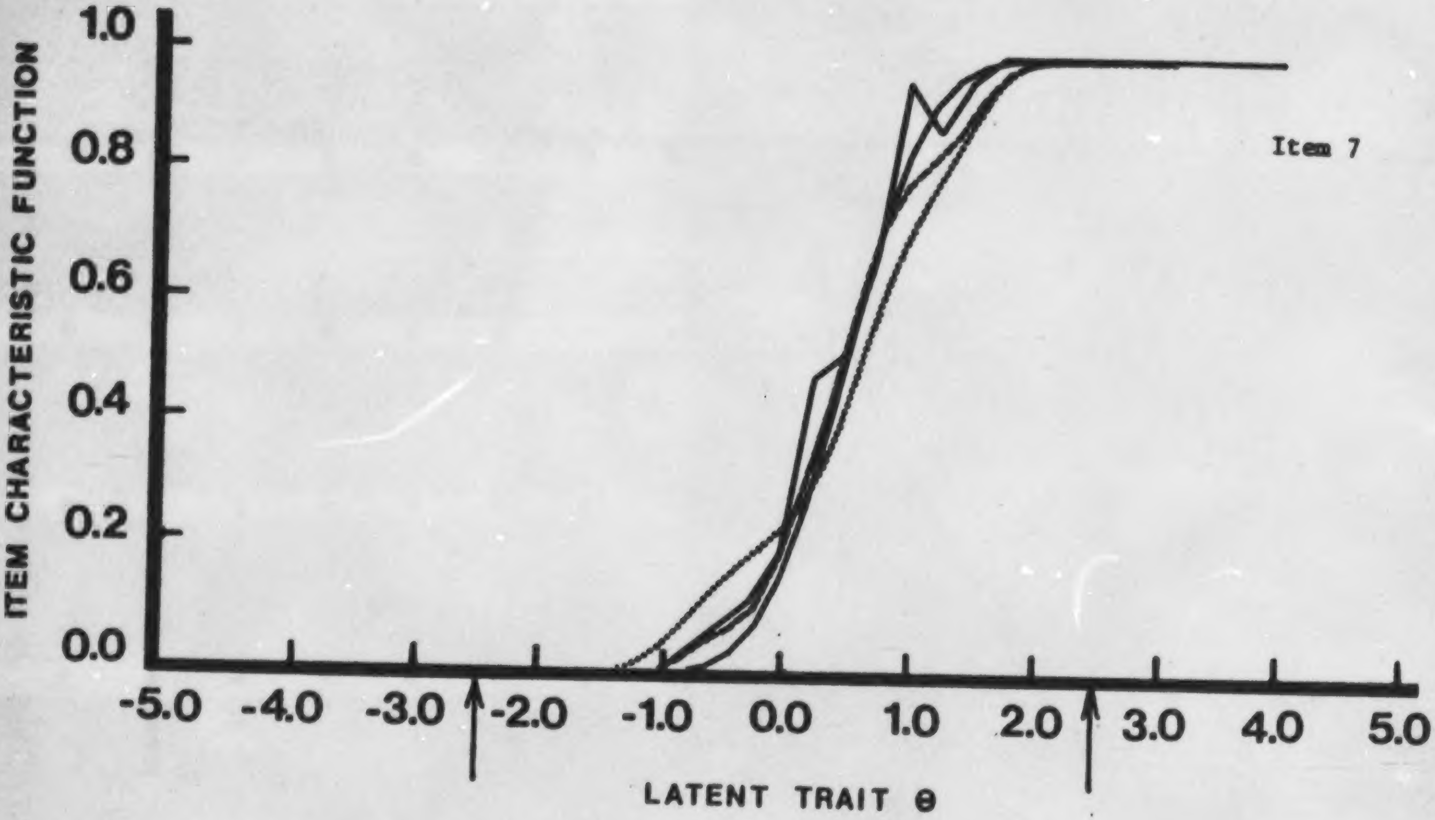


FIGURE 2-7 (Continued): Subtest 2, Degree 3 Case.

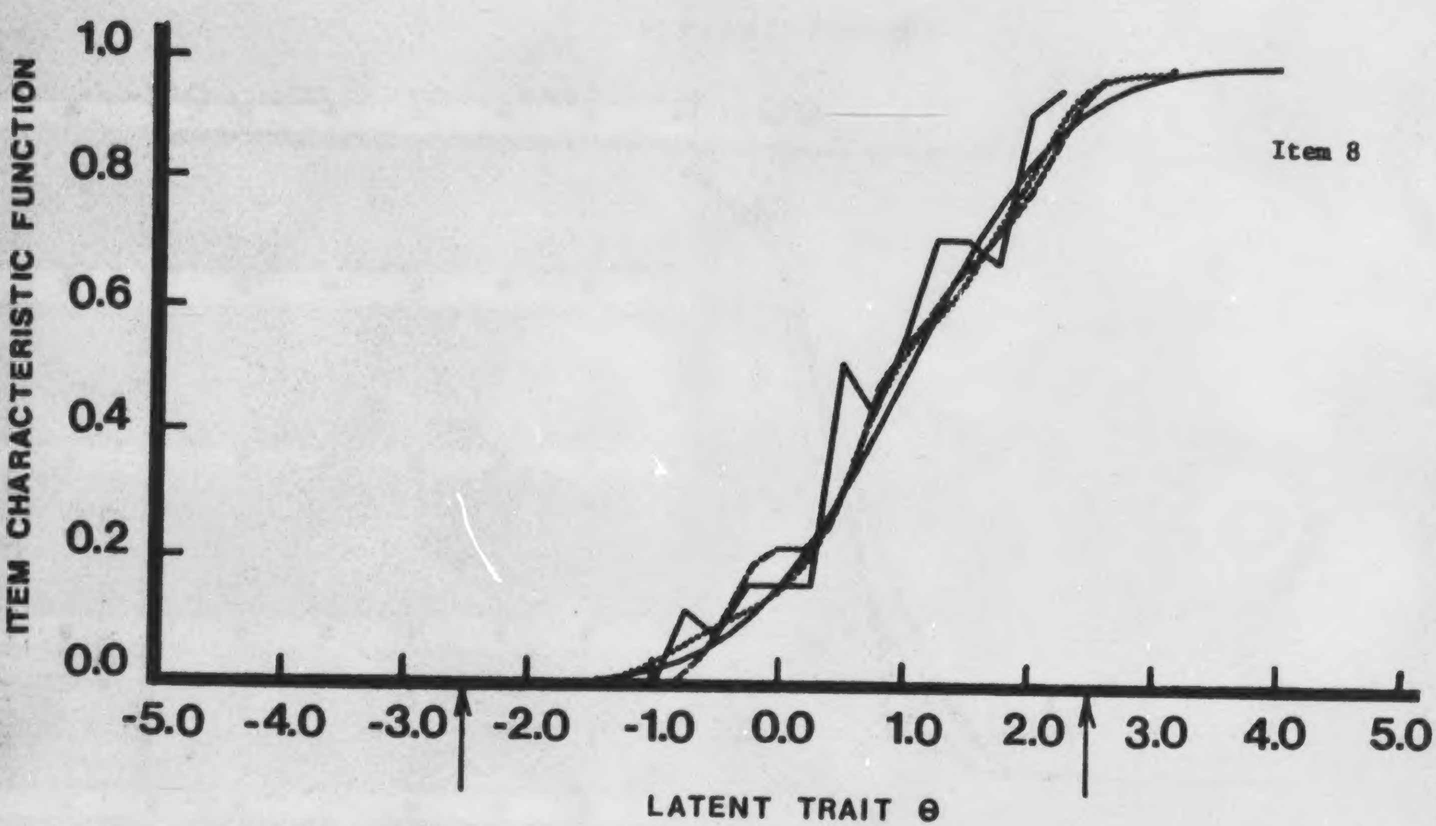


FIGURE 2-7 (Continued): Subtest 2, Degree 3 Case.

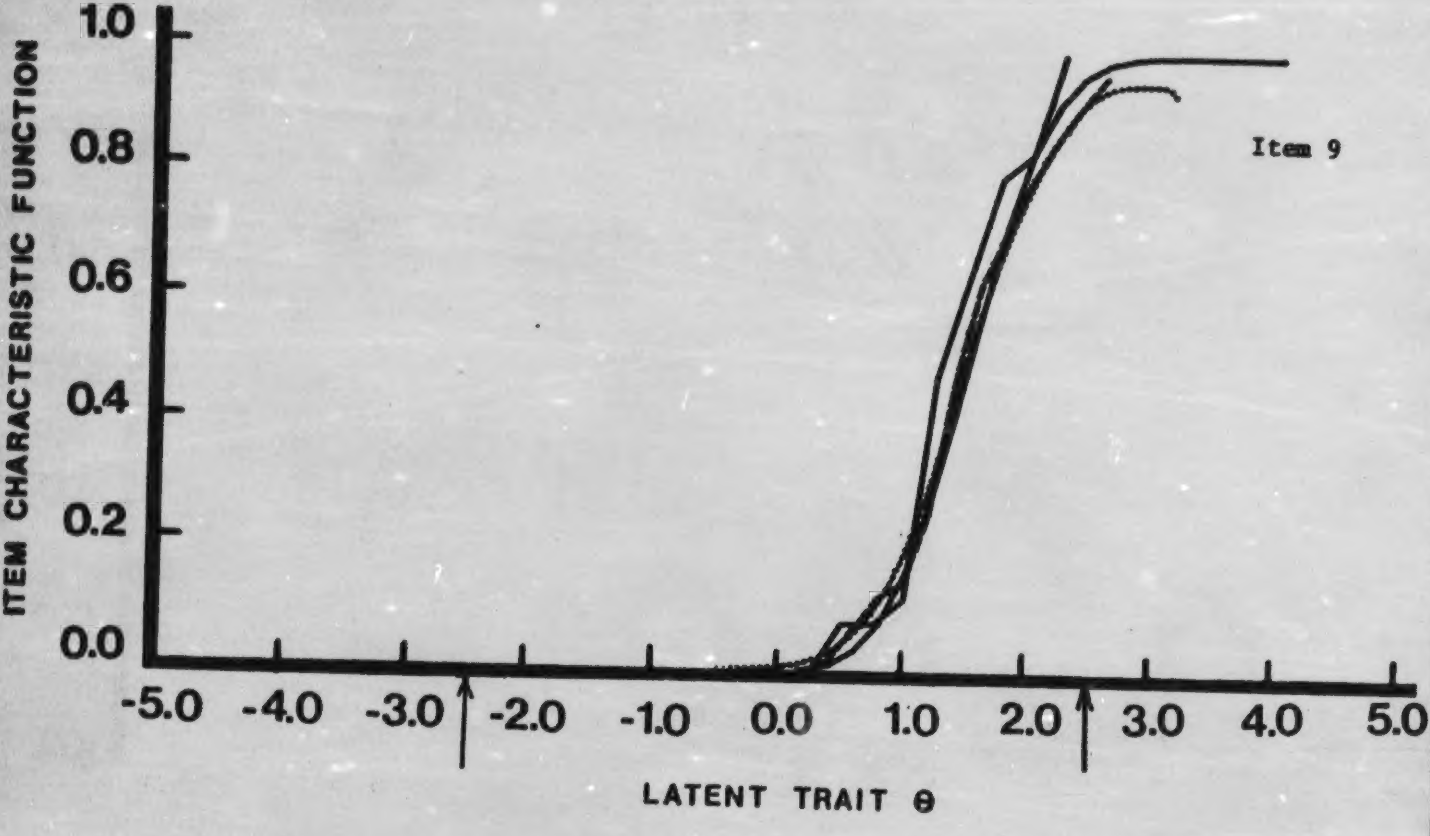


FIGURE 2-7 (Continued): Subtest 2, Degree 3 Case.

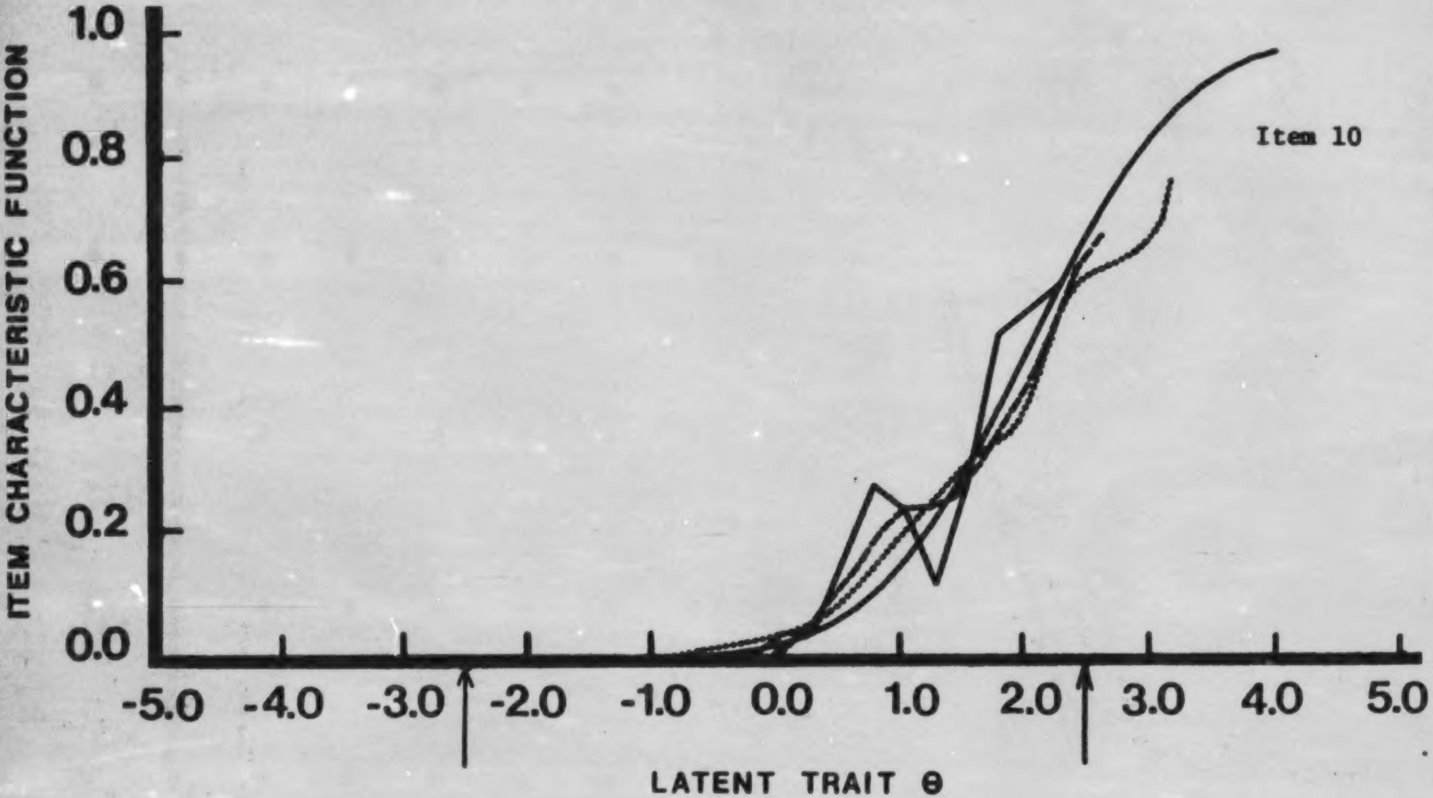


FIGURE 2-7 (Continued): Subtest 2, Degree 3 Case.

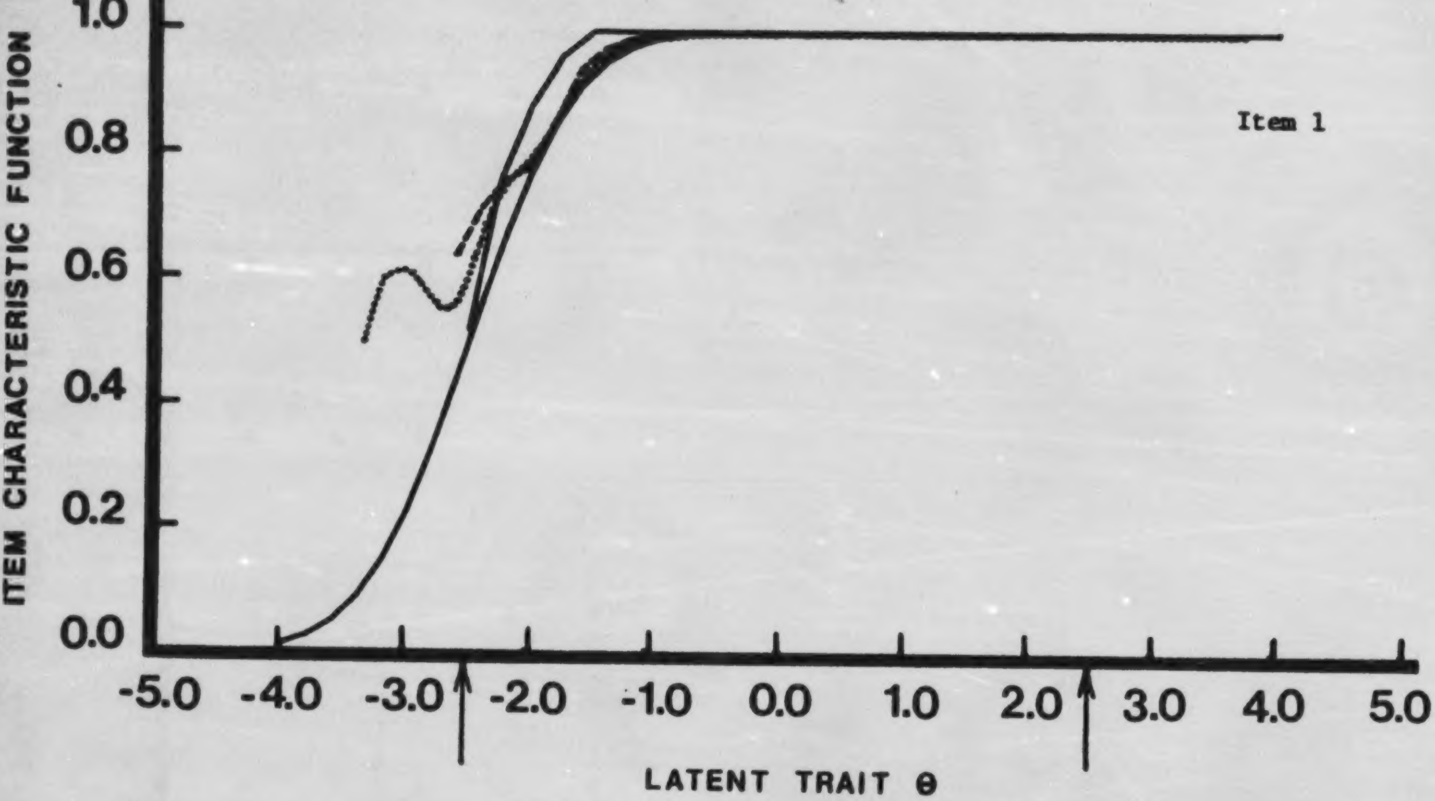


FIGURE 2-8

Estimated Item Characteristic Function Based Upon Subtest 2 (Dotted Curve), in Comparison with the One Based Upon the Total Old Test (Dashed Curve), Obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method. Theoretical Item Characteristic Function (Smooth Solid Curve) and Frequency Ratios of the Correct Answer (Jagged Solid Curve) are Also Presented.
Degree 4 Case

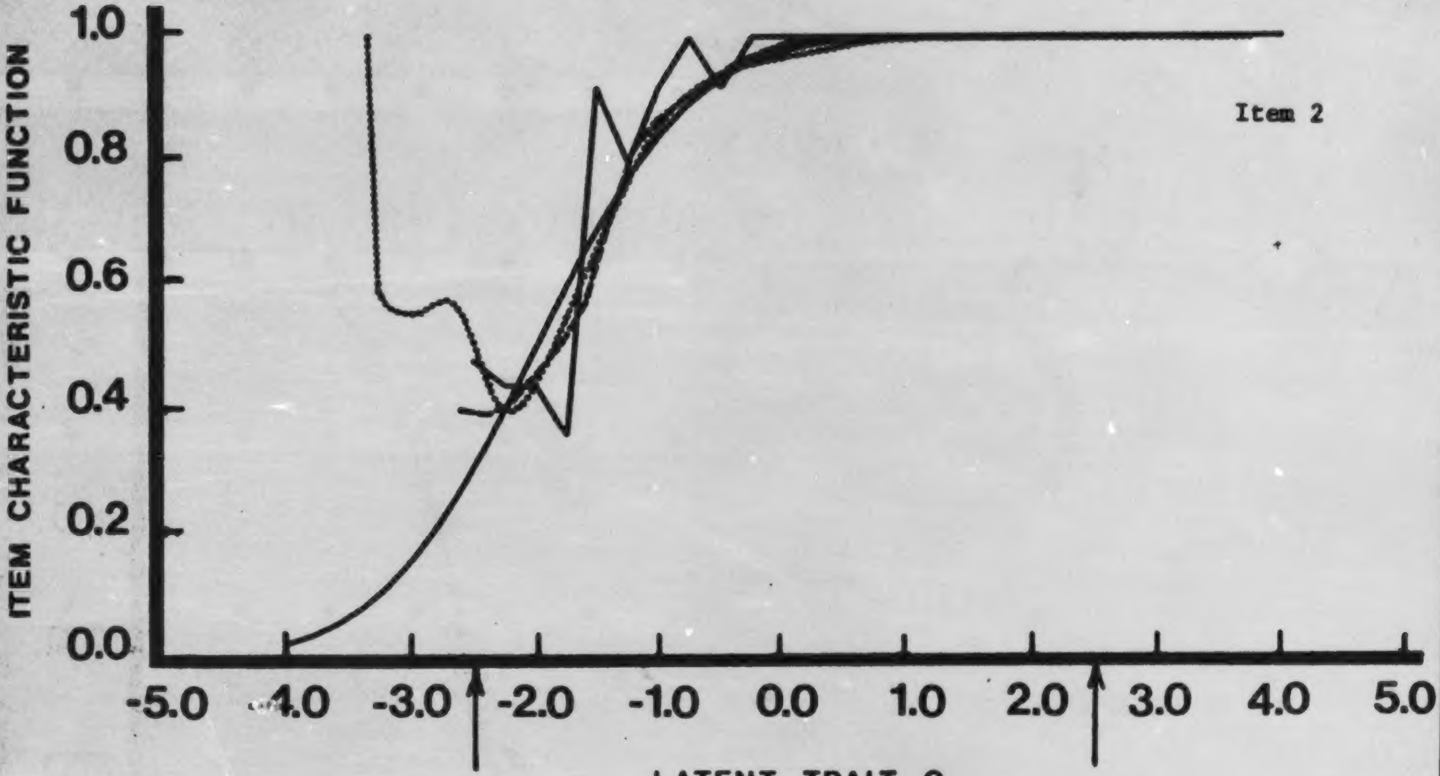


FIGURE 2-8 (Continued): Subtest 2, Degree 4 Case.

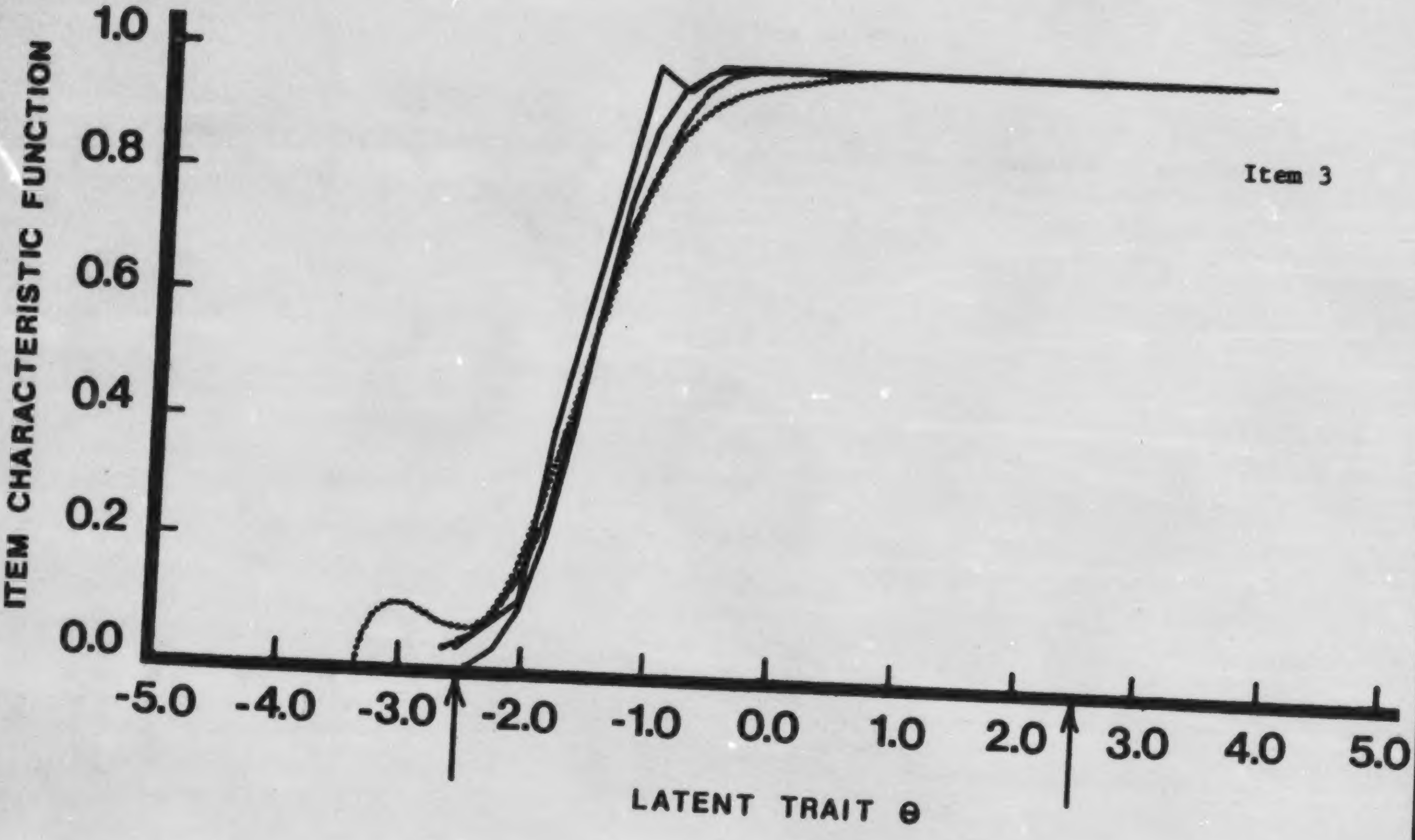


FIGURE 2-8 (Continued): Subtest 2, Degree 4 Case.

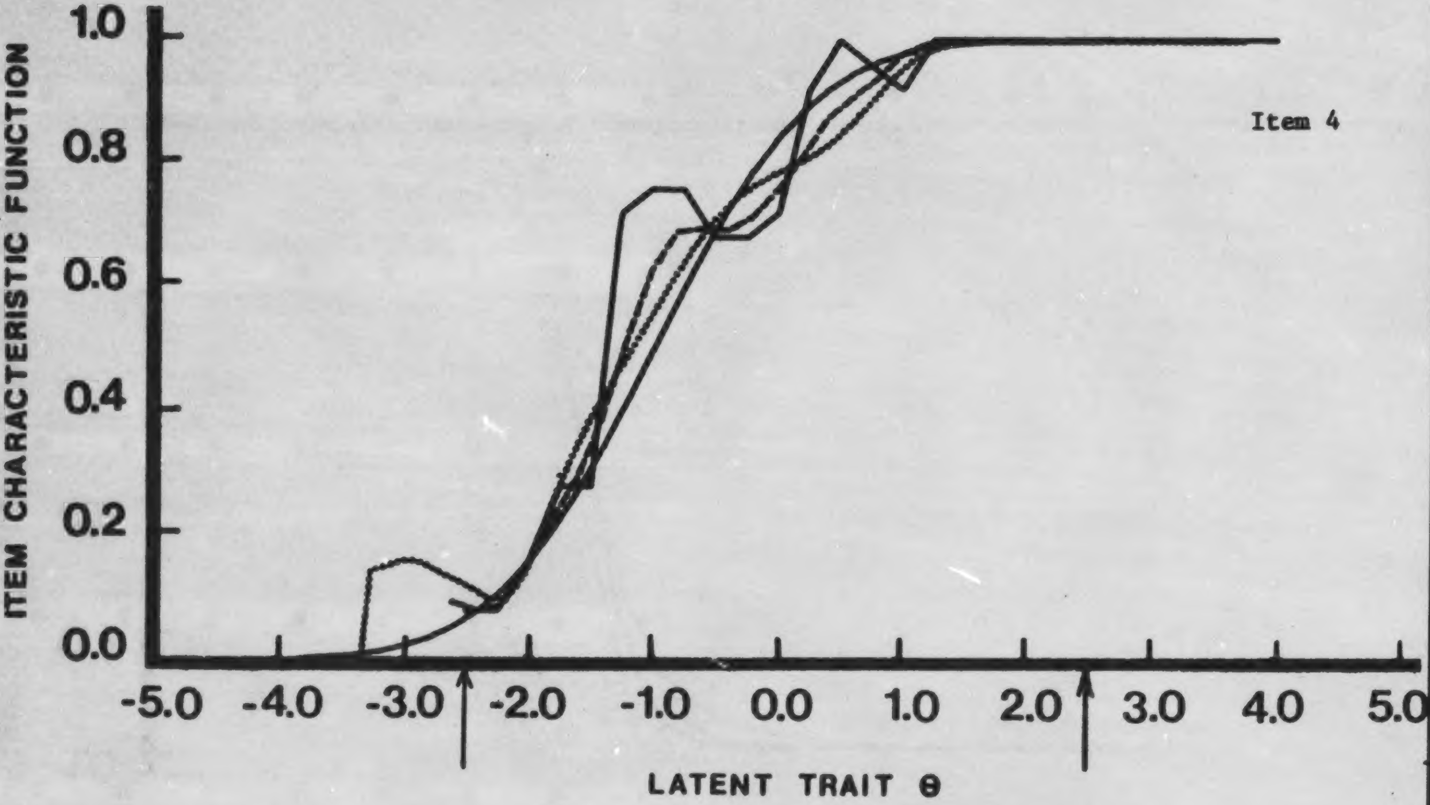


FIGURE 2-8 (Continued): Subtest 2, Degree 4 Case.

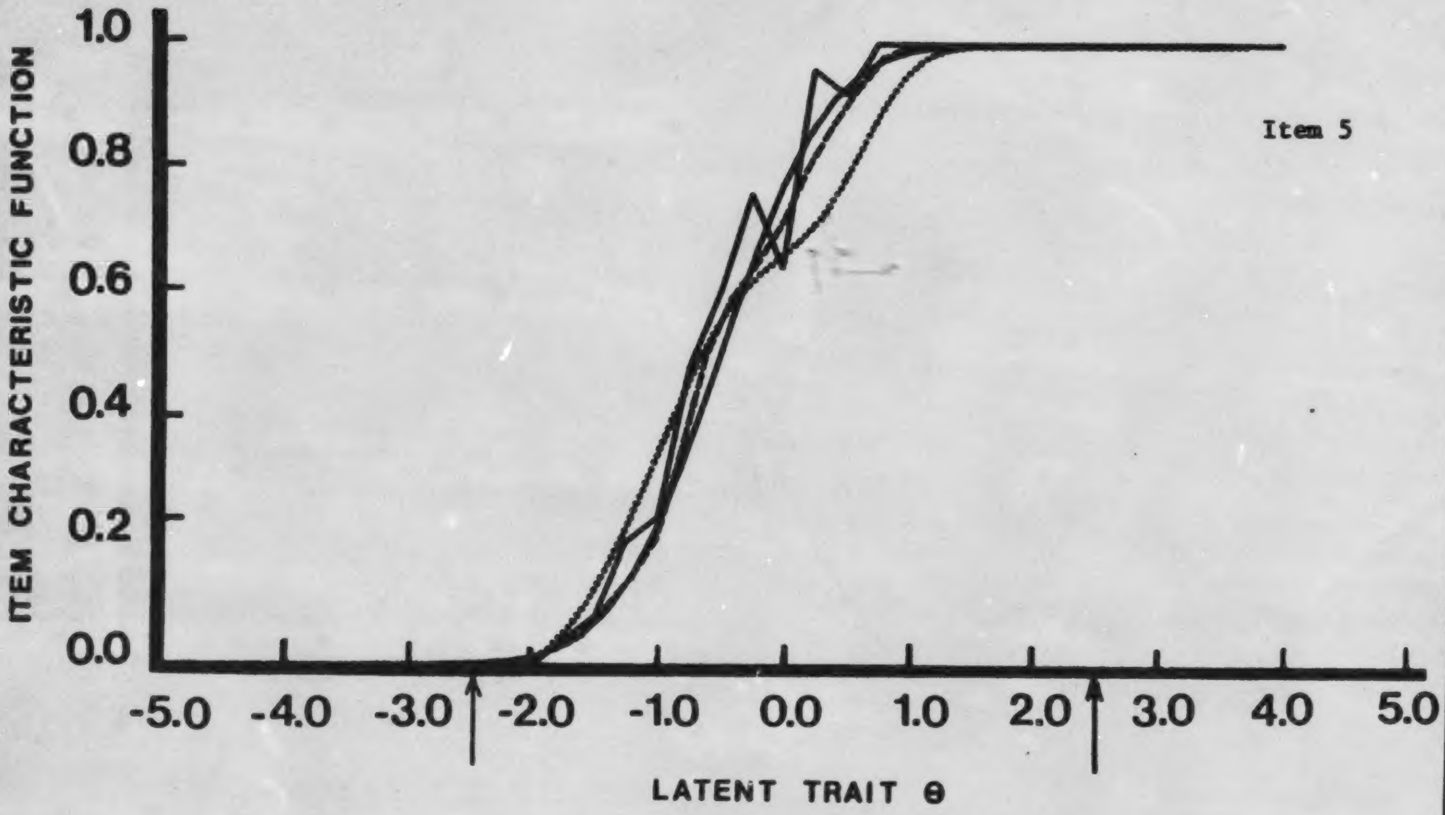


FIGURE 2-8 (Continued): Subtest 2, Degree 4 Case.

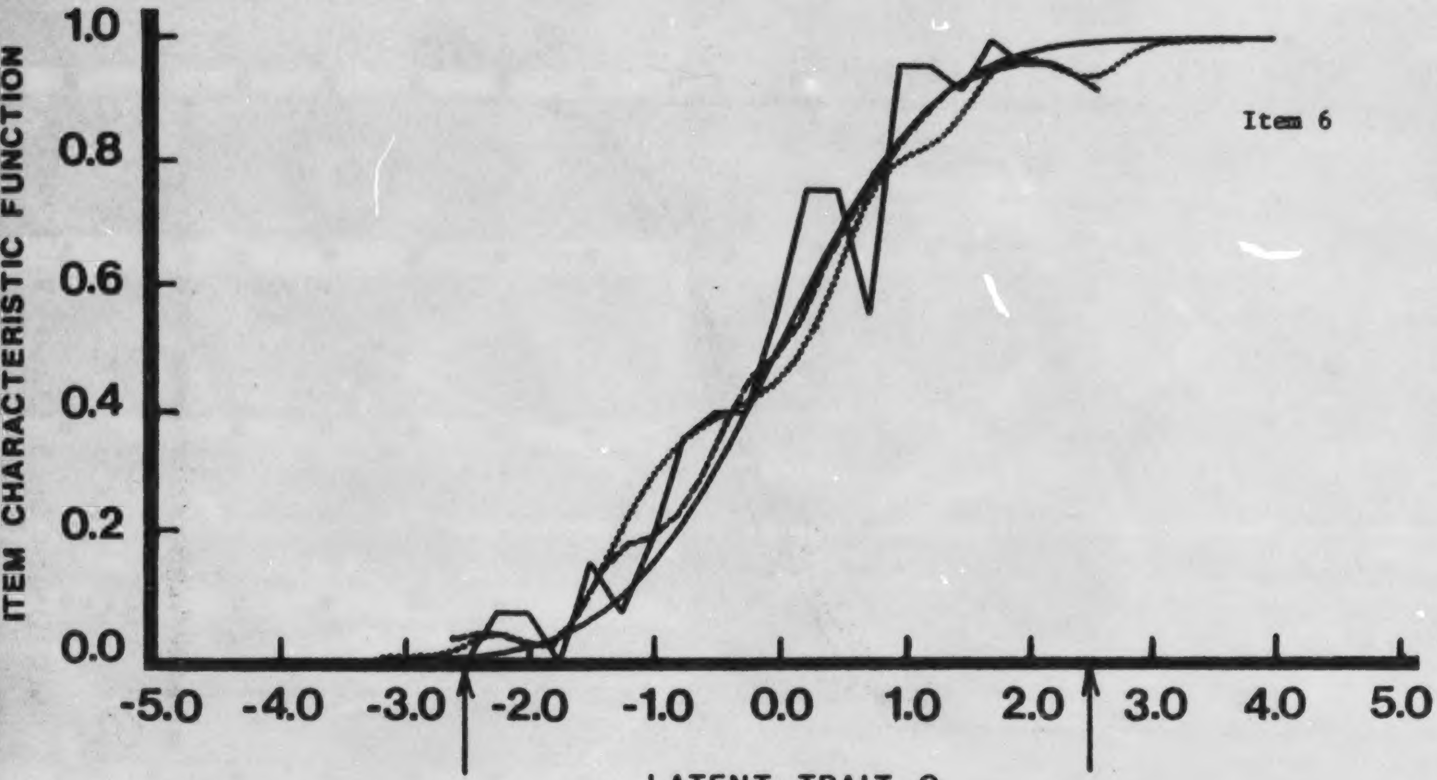


FIGURE 2-8 (Continued): Subtest 2, Degree 4 Case.

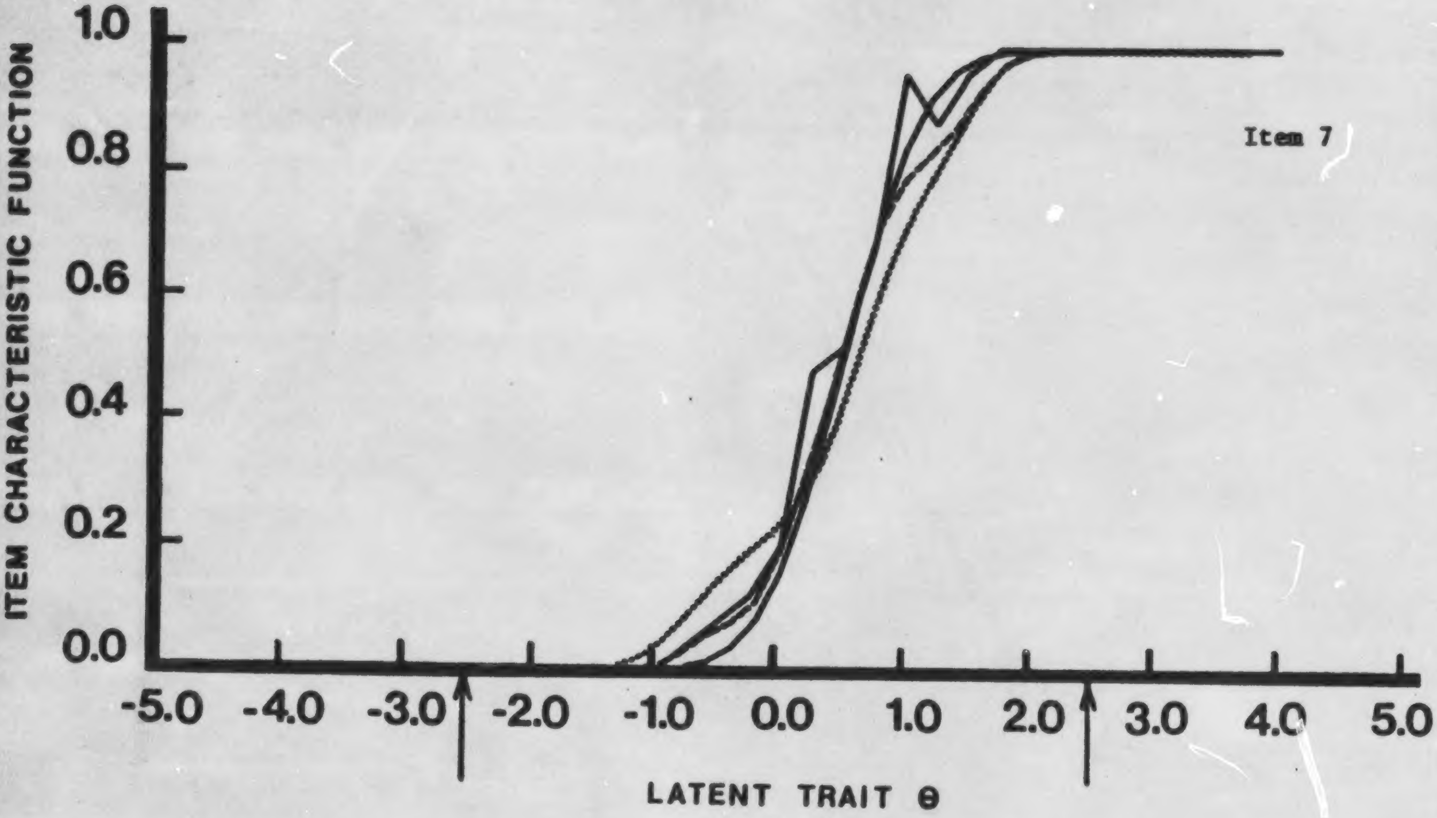


FIGURE 2-8 (Continued): Subtest 2, Degree 4 Case.

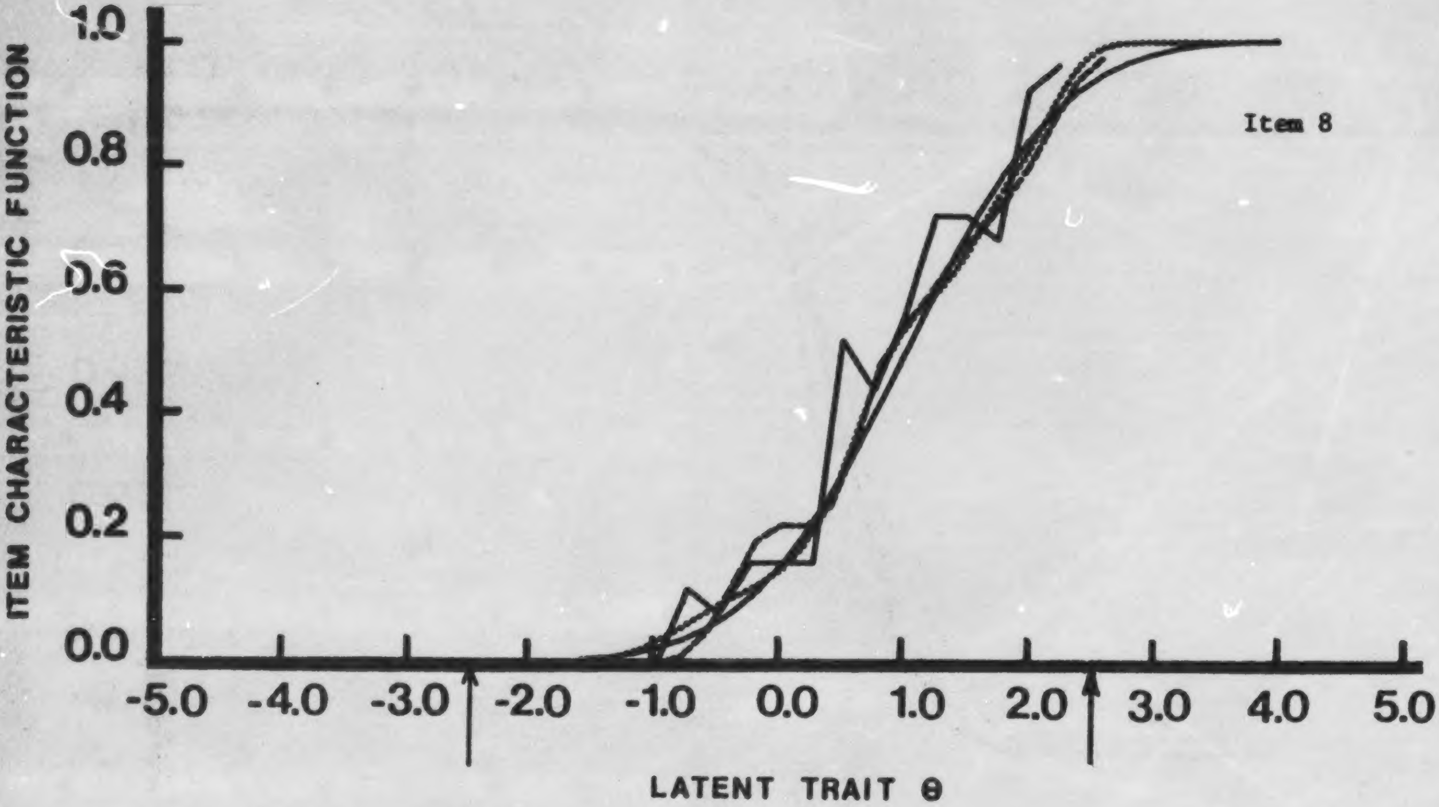


FIGURE 2-8 (Continued): Subtest 2, Degree 4 Case.

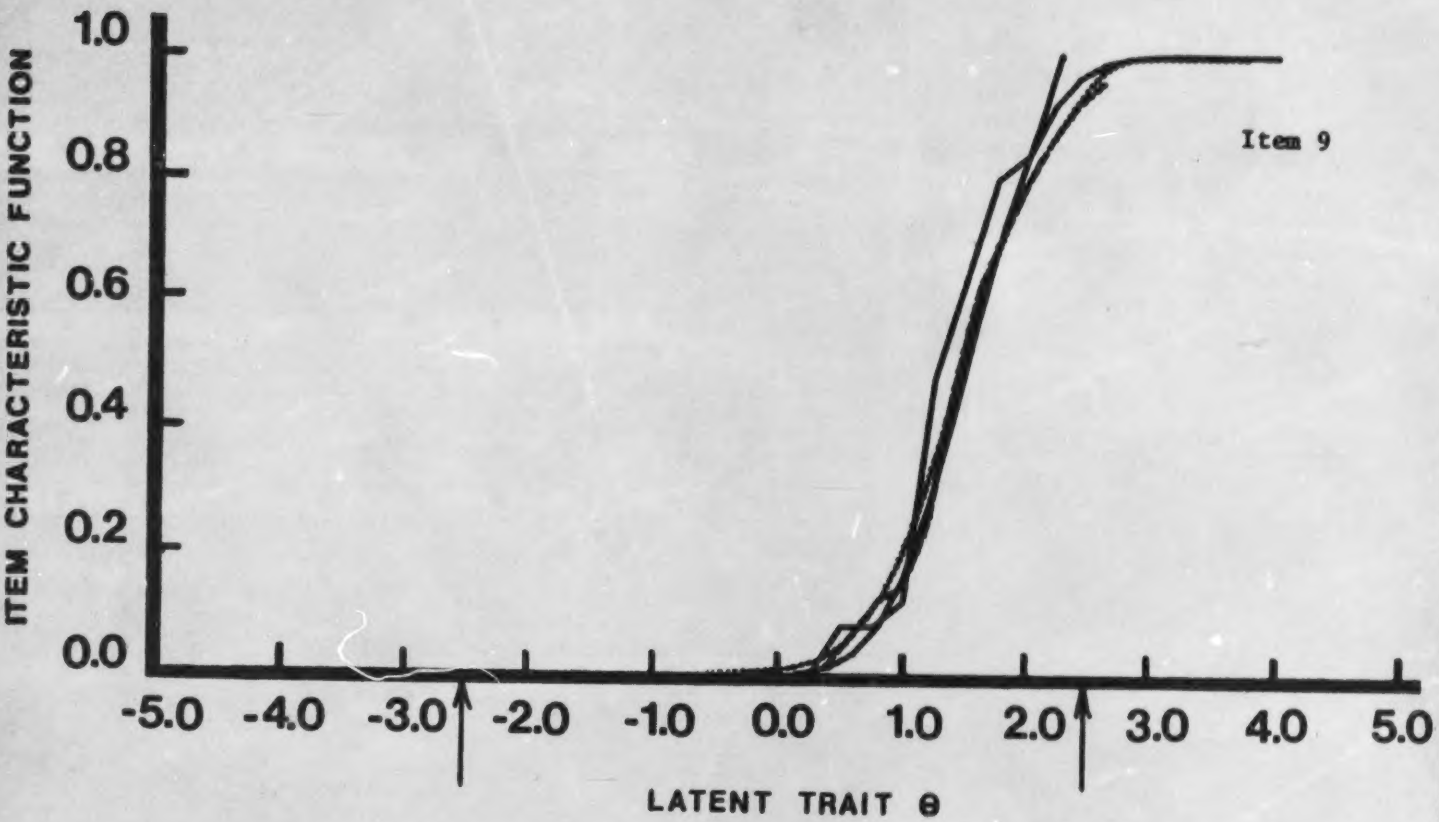


FIGURE 2-8 (Continued): Subtest 2, Degree 4 Case.

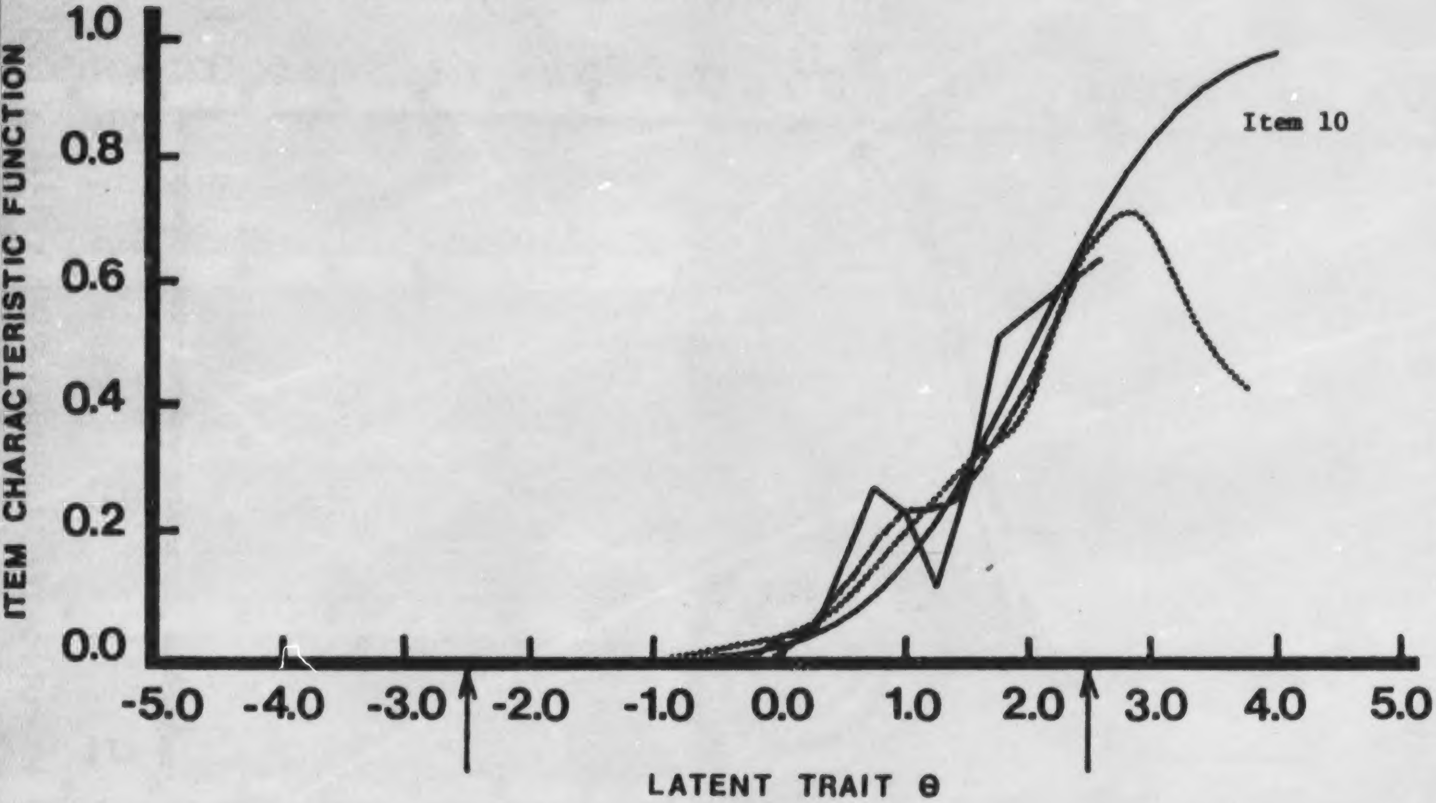


FIGURE 2-8 (Continued): Subtest 2, Degree 4 Case.

III Effect of the Fitness of the Polynomial to the Square Root of the Test Information Function upon the Accuracy of Estimation of the Operating Characteristics

In the preceding chapter, we have seen that the resulting estimates of the ten item characteristic functions based upon Subtest 1 provide us with practically identical functions with those obtained upon the original Old Test, and, therefore, identical with the criterion item characteristic functions, whereas those which are based upon Subtest 2 are somewhat different, in both Degree 3 and 4 Cases. Although all the estimated item characteristic functions, which are based upon Subtest 2, are still good approximations to the corresponding theoretical item characteristic functions, it will be worthwhile to investigate the reason why the differences have resulted.

If we compare the square roots of the test information functions and their approximations in Figures 2-1 and 2-2, we will notice that there are the following two apparent differences between them.

(1) The square root of the test information function of Subtest 2 is substantially different from that of the original Old Test, which is approximately constant, while that of Subtest 1 is relatively close, if we take the interval of θ in which the ability levels of all the five hundred hypothetical examinees are located, i.e., $(-2.5, 2.5)$.

(2) For Subtest 2, the polynomial of degree 7, which was obtained by the method of moments with $[-4.0, 4.0]$ as the interval of θ, is a relatively poor approximation to the square root of the test information function, in comparison with the one for Subtest 1.

It is recalled that the substitution of the polynomial for the square root of the test information function has simplified the process of transformation of θ to τ, and also that of retransformation of τ to θ. If the discrepancies caused by this approximation substantially affects the resulting estimates of the item characteristic functions, however, we will have to find a way of correcting them.

We can write

$$[I^*(\tau)]^{1/2} = [I(\theta)]^{1/2} \frac{d\theta}{d\tau} \quad , \tag{3.1}$$

where $I^*(\tau)$ is the test information function defined after the transformation of θ to τ. The derivative of θ with respect to τ is obtained from (2.7) and (2.8) such that

$$\frac{d\theta}{d\tau} = [\frac{d\tau}{d\theta}]^{-1} = [\sum_{k=1}^{m+1} k\, \alpha_k^* \theta^{k-1}]^{-1} \tag{3.2}$$

$$= C[\sum_{k=0}^{m} \alpha_k \theta^k]^{-1} \quad .$$

We recall that our rationale is based upon the constancy of the square root of the test information defined after the transformation of θ to τ , i.e., the left hand side of (3.1). Thus we can say that, if by substituting the rightest hand side of (3.2) into (3.1) we obtain a function which is substantially different from C , those differences may be a reason for the discrepancies in the resulting estimated item characteristic functions from the criterion item characteristic functions. Figures 3-1 and 3-2 present these functions for Subtests 1 and 2, respectively, in comparison with C , which is drawn by a solid line in each figure. We can see in these results that for Subtest 2 the resulting function is substantially different from C $(= 3.5)$, especially for the values of τ between -1.5 and 0.5 , whereas for Subtest 1 it is practically identical with C $(= 4.5)$. We must suspect, therefore, that these large discrepancies from the constancy for Subtest 2 might be a reason for the relatively poor estimation of the item characteristic functions.

We have seen (Samejima and Livingston, 1979) that polynomials obtained by the method of moments are substantially affected by the interval of the independent variable we take, as is obvious from the fact that they are the least squares solutions with respective degrees. For this reason, we obtained several sets of polynomials of degrees 3 through 7 for Subtest 2, by changing the interval of θ in the process of applying the method of moments for the square root of the test information function. These results are shown for the five different intervals of θ in Appendix I. As we can see, in those

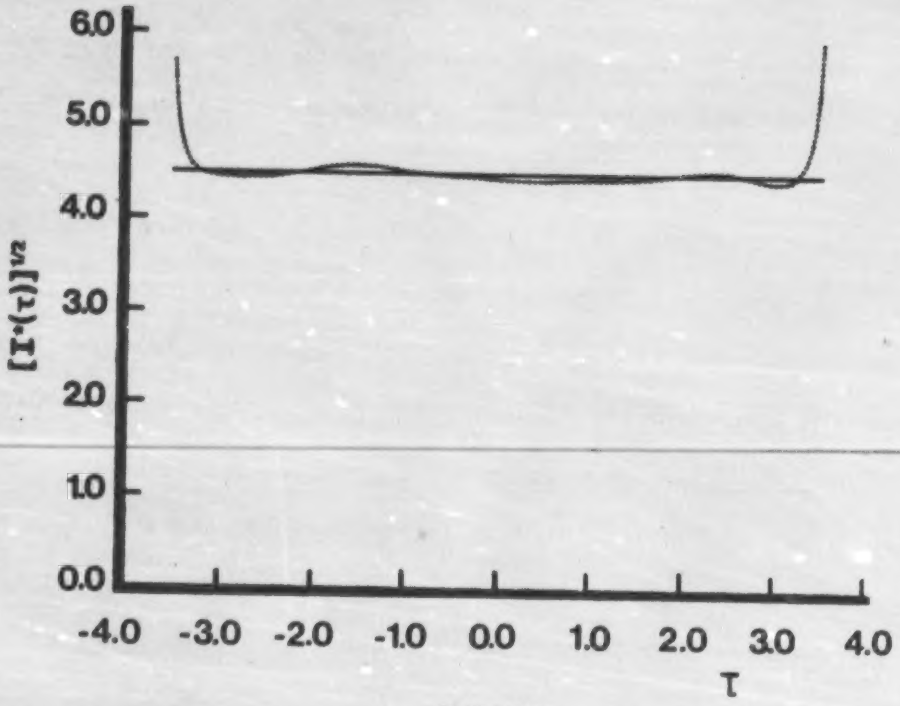


FIGURE 3-1

Square Root of the Constant Test Information, $[I^{*}(\theta)]^{1/2}$, of Subtest 1 (Dotted Curve) Using the Polynomial of Degree 8 in Transforming θ to τ, in Comparison with Its Theoretical Values (Solid Line).

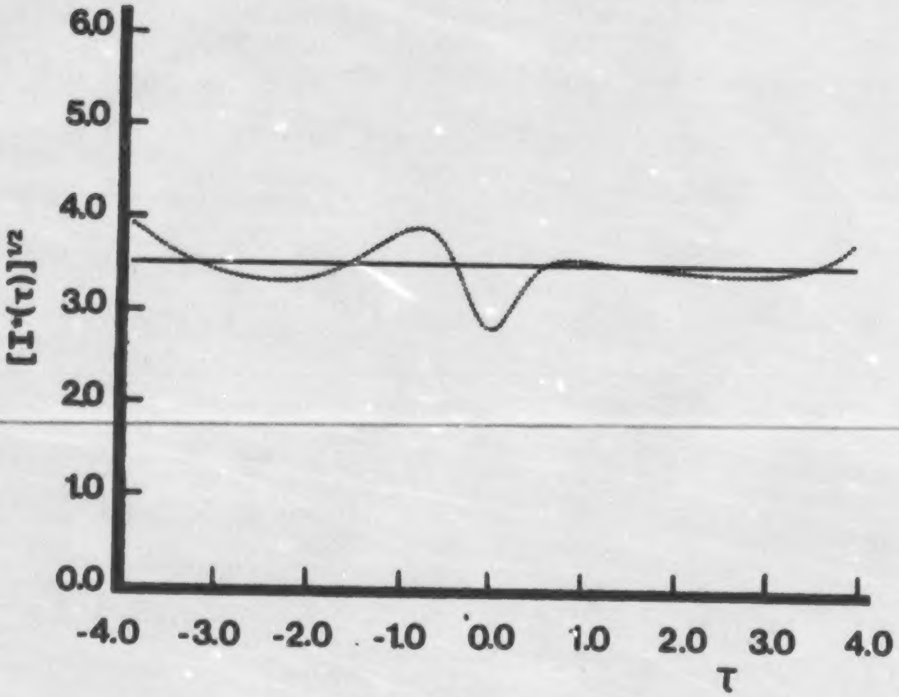


FIGURE 3-2

Square Root of the Constant Test Information, $[I^*(\theta)]^{1/2}$, of Subtest 2 (Dotted Curve) Using the Polynomial of Degree 8 in Transforming θ to τ, in Comparison with Its Theoretical Values (Solid Line).

results, that none of them provides us with a better approximation to the original function. This is obviously caused by the fact that the square root of the test information function for Subtest 2 has a more complicated, bimodal shape, unlike the one for Subtest 1. We must change our strategy, therefore, and divide the interval of θ into several subintervals, and try to fit a polynomial to each segment of the function.

After some trials and errors, we have found a set of polynomials which fit well to the square root of the test information function of Subtest 2, by using three subintervals, $[-6.0, -1.5]$, $[-1.5, 0.5]$ and $[0.5, 6.0]$. The resulting three polynomials are:

$$(3.3) \quad \begin{cases} 2.322 + 0.076\theta + 1.074\theta^2 + 0.352\theta^3 + 0.028\theta^4 \\ \qquad\qquad\qquad \text{for} \quad -6.0 \le \theta < -1.5 \\ 1.760 + 0.314\theta + 1.517\theta^2 - 0.190\theta^3 - 0.254\theta^4 + 0.054\theta^5 \\ \qquad\qquad\qquad \text{for} \quad -1.5 \le \theta < 0.5 \\ 1.352 + 2.041\theta - 0.184\theta^2 - 0.070\theta^3 + 0.070\theta^4 \\ \qquad\qquad\qquad \text{for} \quad 0.5 \le \theta \le 6.0 \end{cases} .$$

Figure 3-3 presents these polynomials plotted by dotted curves in comparison with the square root of the test information function of Subtest 2, which is drawn by a solid curve. As we can see, the approximations of these three curves to the original function within the respective subintervals are far better than the approximation obtained by the single polynomial of degree 7, which is shown in Figure 2-2. From these coeficients, three sets of transformations

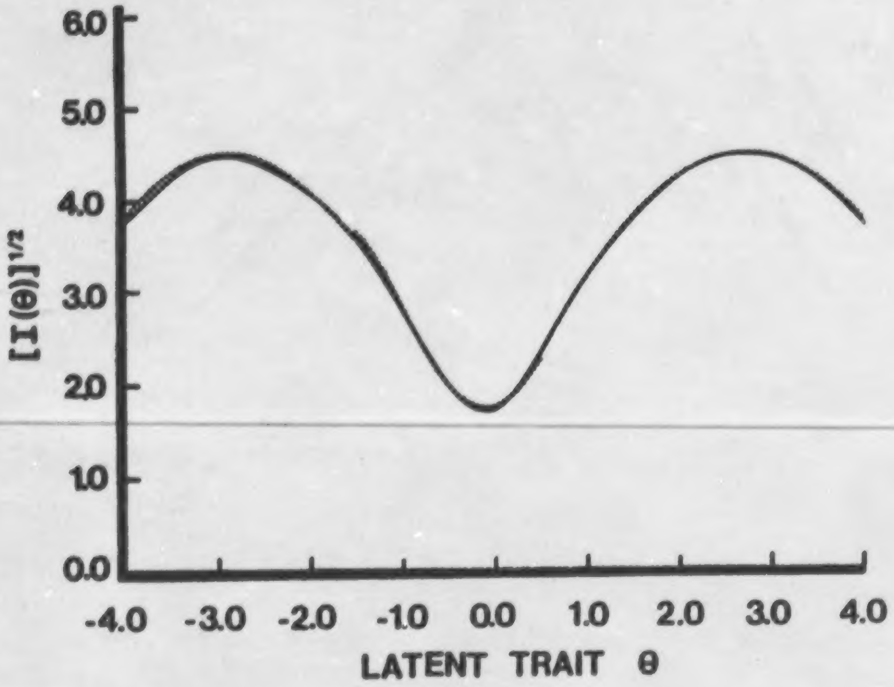


FIGURE 3-3

Three Polynomials (Dotted Curves) Approximating the Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Curve) of Subtest 2 .

of θ to τ were obtained through (2.7) and (2.8), using $d = 0$, such that

$$(3.4) \quad \begin{cases} \tau = 0.66343\theta + 0.01086\theta^2 + 0.10229\theta^3 + 0.02514\theta^4 + 0.00160\theta^5 \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } -1.5 < \theta \\ \tau = 0.50286\theta + 0.04486\theta^2 + 0.14448\theta^3 - 0.01357\theta^4 \\ \qquad\qquad - 0.01451\theta^5 + 0.00257\theta^6 \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } -1.5 \le \theta < 0.5 \\ \tau = 0.38629\theta + 0.29157\theta^2 - 0.01752\theta^3 - 0.00500\theta^4 + 0.00040\theta^5 \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \theta \le 0.5 \end{cases} .$$

The five hundred maximum likelihood estimates, $\hat{\theta}$, were classified into three subgroups, depending upon which of the above three subintervals each maximum likelihood estimate belongs to. The resulting numbers of $\hat{\theta}$'s in the three subgroups are 98, 288 and 214, respectively. Then each maximum likelihood estimate, $\hat{\theta}$, was transformed to $\hat{\tau}$, through (2.11), following the specific transformation assigned to each subgroup.

Figure 3-4 presents the frequency distribution of the five hundred $\hat{\tau}$'s thus obtained, together with the polynomials of degrees 3 through 7, which were obtained by the method of moments based upon the five hundred $\hat{\tau}$'s to provide us with approximated density functions, $\hat{g}(\hat{\tau})$. The coefficients of these five polynomials are presented in Table 3-1.

As before, we used each of the first two polynomials of degrees 3 and 4 for $\hat{g}(\hat{\tau})$ in estimating the item characteristic functions of

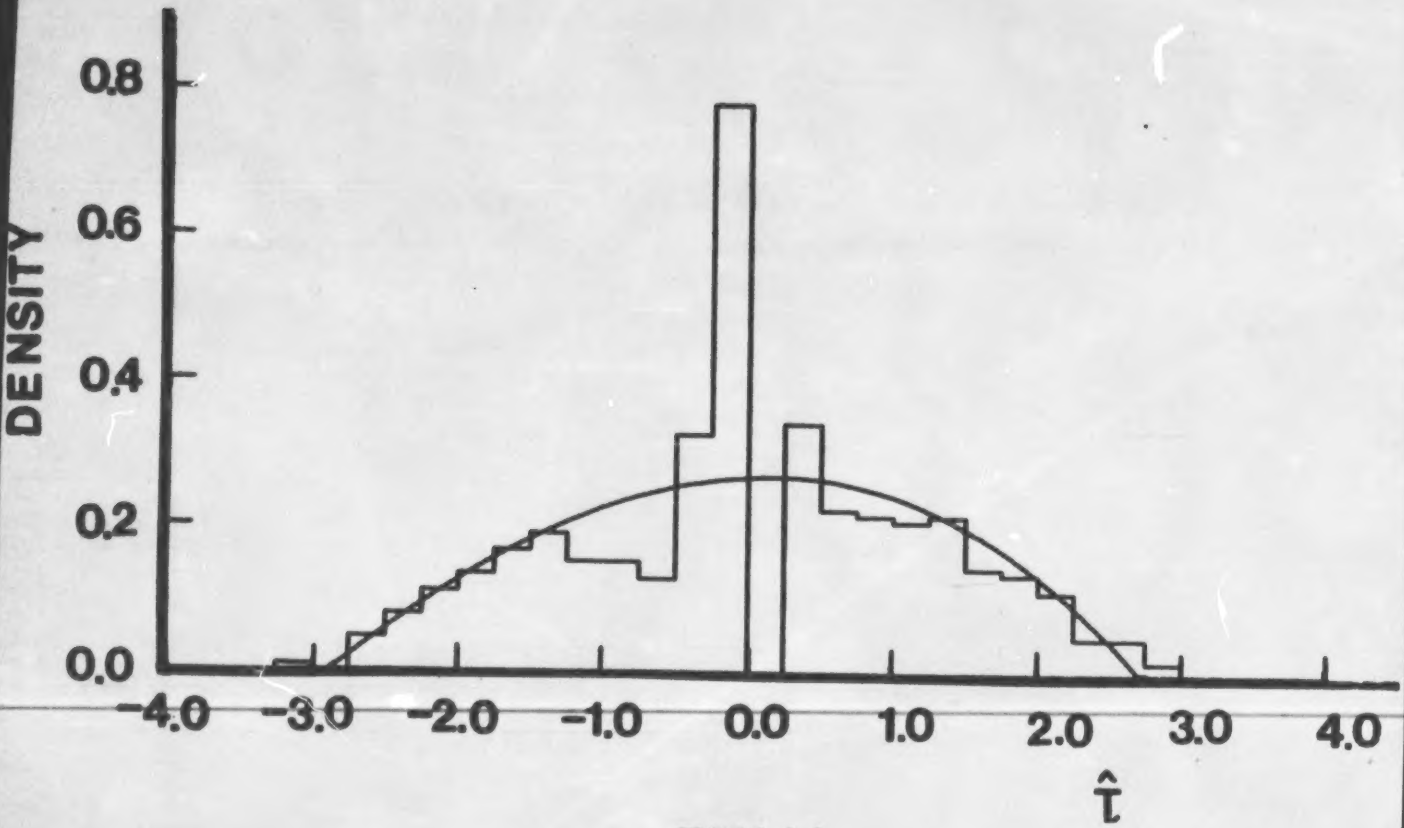


FIGURE 3-4

Relative Frequency Distribution of $\hat{\tau}$ Obtained upon Subtest 2, but Using the Set of Three Different Transformations of $\hat{\theta}$ to $\hat{\tau}$, Together with the Polynomial of Degree 3 Obtained by the Method of Moments to Approximate $g(\hat{\tau})$.

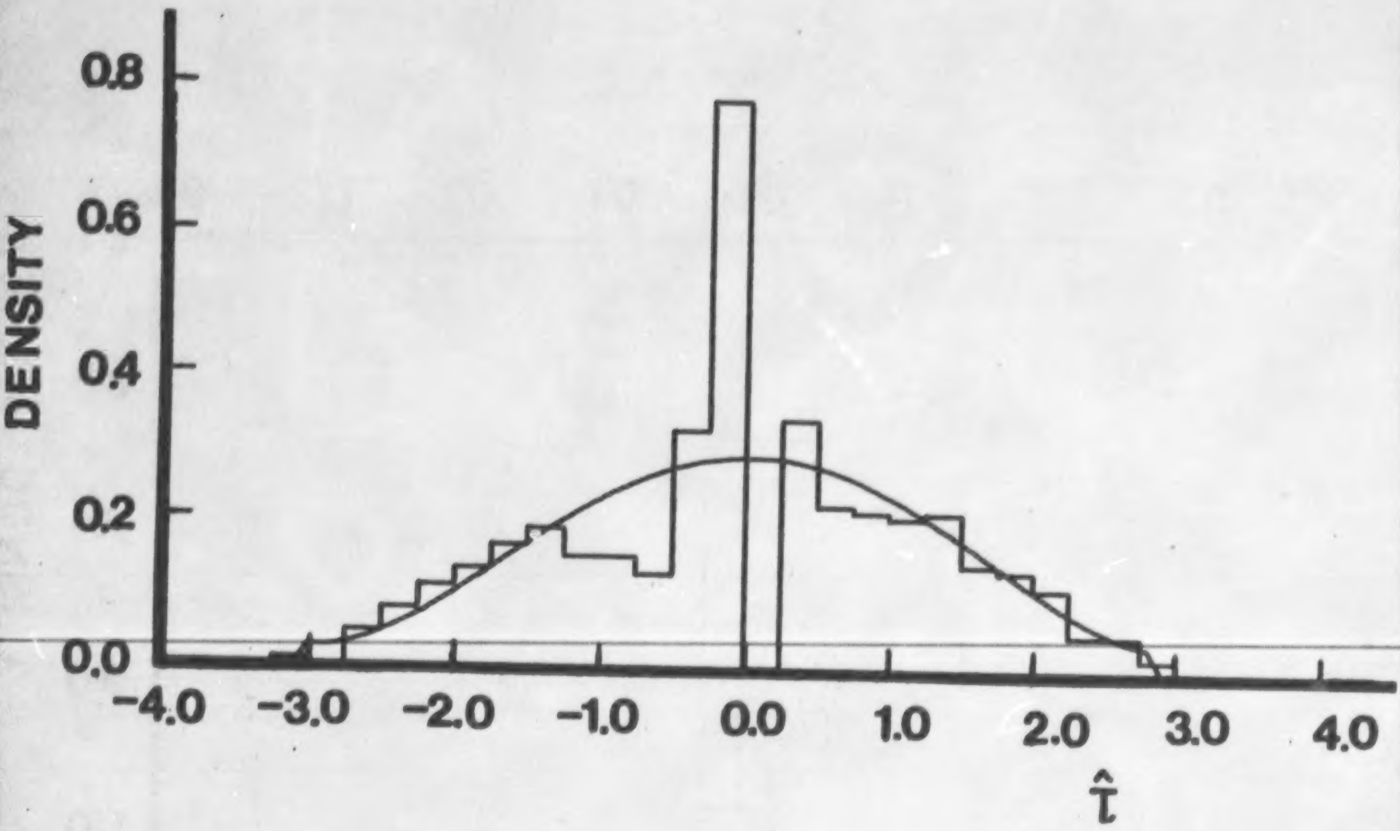


FIGURE 3-4 (Continued): Subtest 2 with Three Different Transformations, Polynomial of Degree 4.

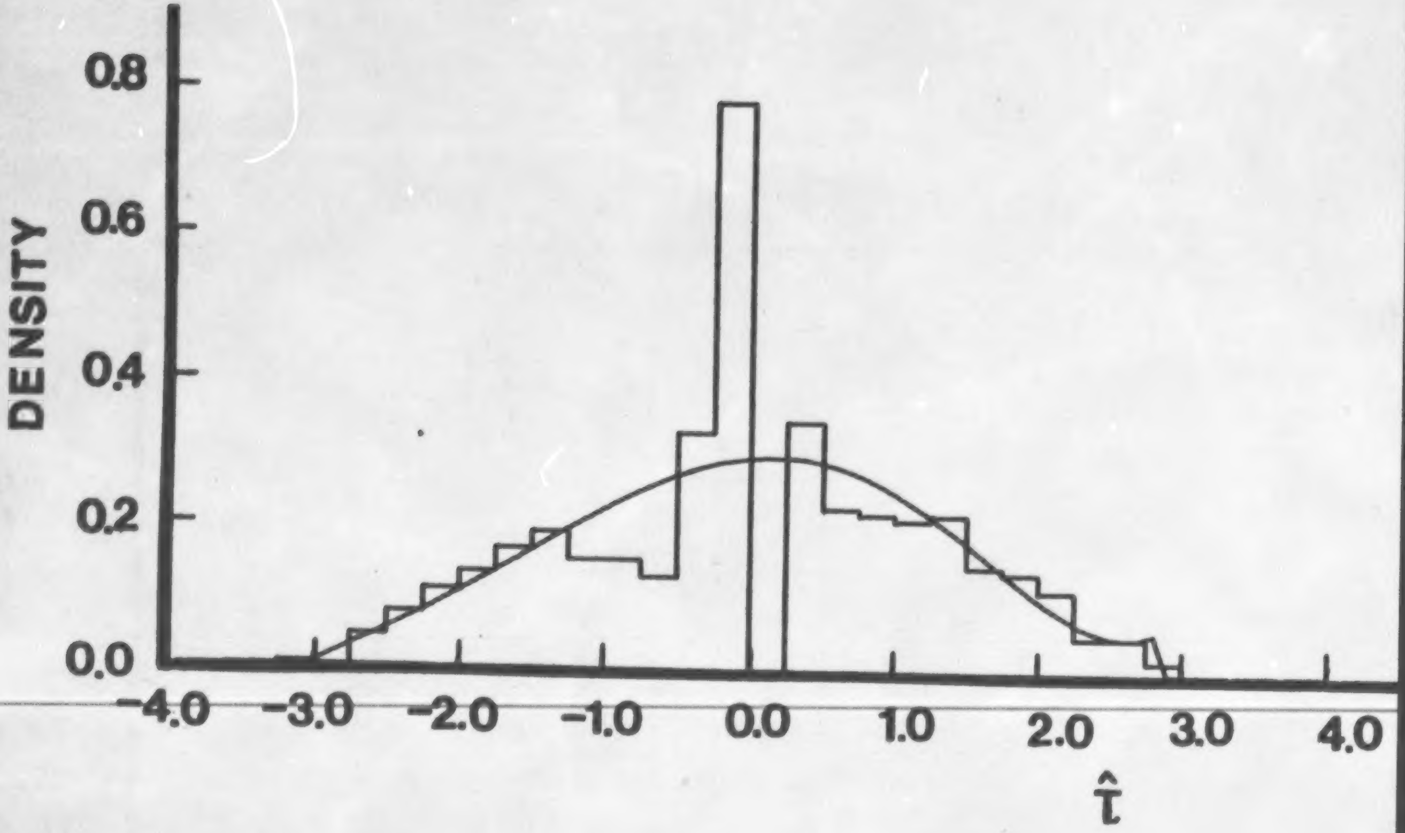


FIGURE 3-4 (Continued): Subtest 2 with Three Different Transformations, Polynomial of Degree 5.

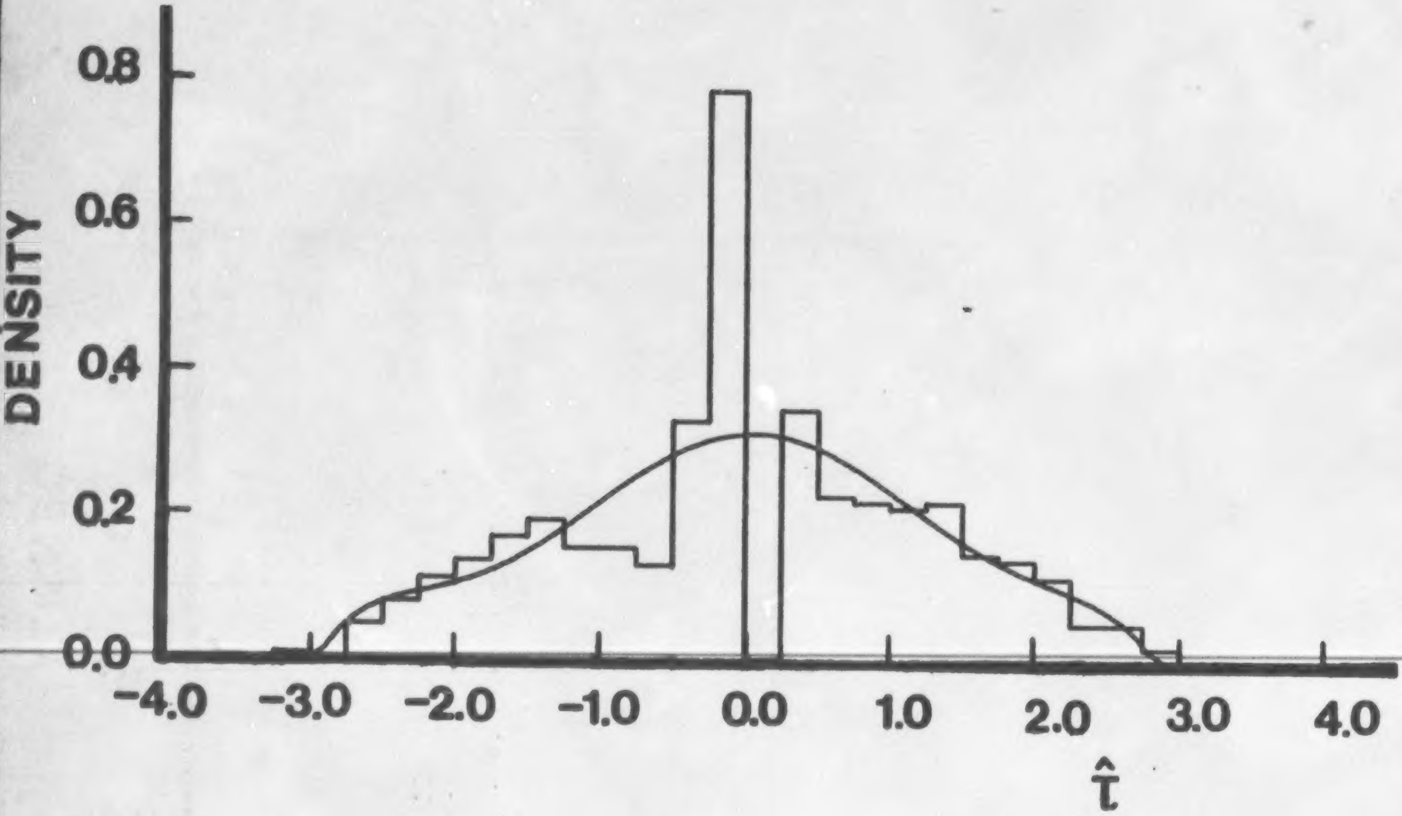


FIGURE 3-4 (Continued): Subtest 2 with Three Different Transformations, Polynomial of Degree 6.

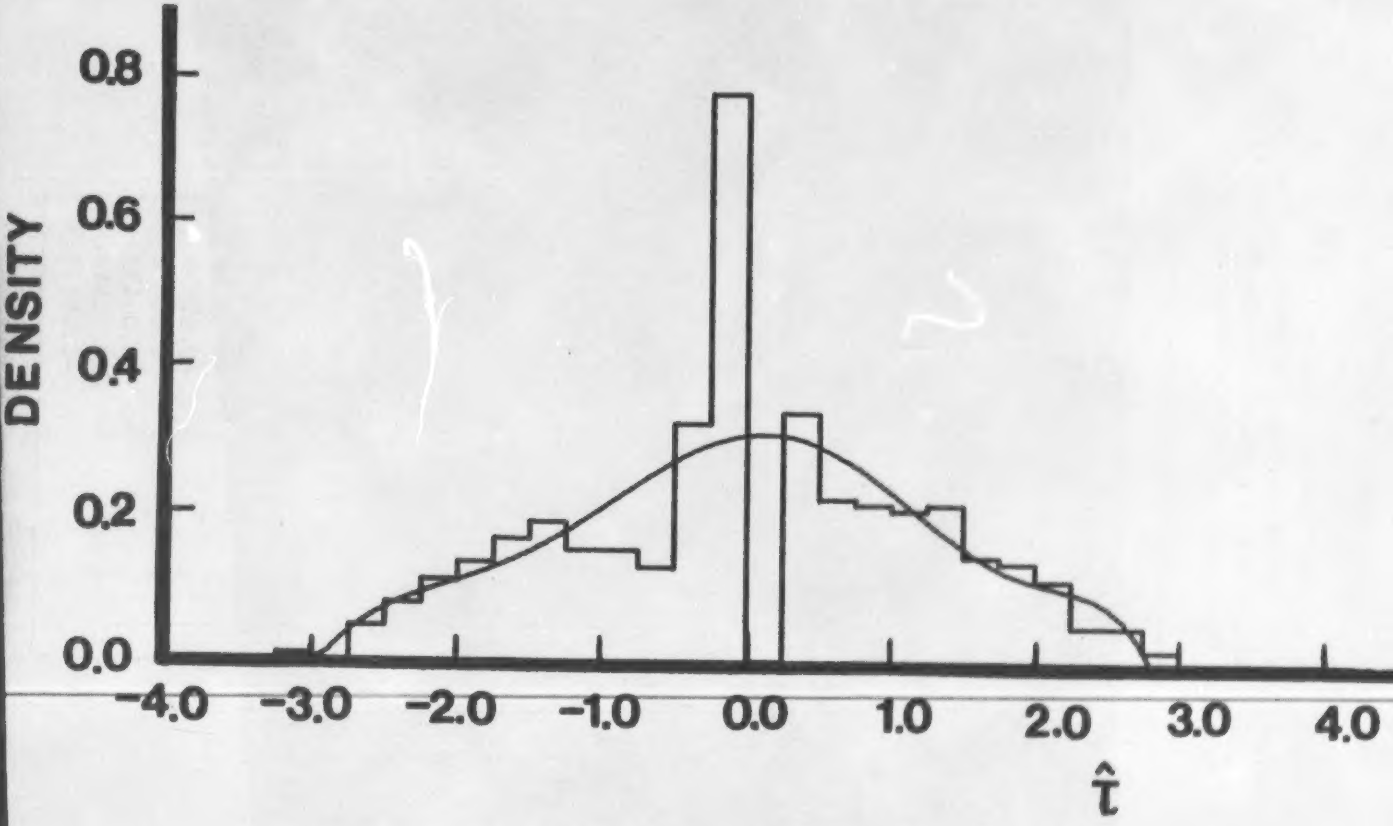


FIGURE 3-4 (Continued): Subtest 2 with Three Different Transformations, Polynomial of Degree 7.

TABLE 3-1

Coefficients of the Five Polynomials of Degrees 3 Through 7 for Approximating the Density Function, $g(\hat{\tau})$, Which Were Obtained by the Method of Moments Upon the Set of Five Hundred $\hat{\tau}$'s .

		Coefficient
DGR. 3	0	0.27022
	1	0.00937
	2	-0.03486
	3	-0.00193
DGR. 4	0	0.29201
	1	0.00338
	2	-0.06002
	3	-0.00032
	4	0.00341
DGR. 5	0	0.29397
	1	0.01971
	2	-0.06138
	3	-0.00915
	4	0.00396
	5	0.00093
DGR. 6	0	0.31083
	1	0.00987
	2	-0.10404
	3	-0.00234
	4	0.01820
	5	0.00006
	6	-0.00122
DGR. 7	0	0.31277
	1	0.02585
	2	-0.11008
	3	-0.01899
	4	0.02076
	5	0.00433
	6	-0.00148
	7	-0.00031

the ten binary items through the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method. Figures 3-5 and 3-6 present these resulting estimates of the item characteristic functions by dotted curves, in comparison with those obtained upon the original Old Test, together with the corresponding theoretical item characteristic functions and the frequency ratios of the correct answers, in Degree 3 and 4 Cases, respectively.

As we can see in these graphs, in both Degree 3 and 4 Cases, the results are very similar to those obtained by approximating the square root of the test information function of Subtest 2 by the single polynomial of degree 7, which are shown as Figures 2-7 and 2-8, respectively. It is also noticed that the results of Degree 3 and 4 Cases are very similar to each other for every binary item, as was the case with all the previous results.

We must conclude, therefore, that improvement in the approximation to the square root of the test information function of Subtest 2 has little to do with the accuracy of estimation of the item characteristic functions. The discrepancies of these estimated item characteristic functions from the corresponding criterion item characteristic functions, which were obtained for the original Old Test, must be caused by some other reasons.

The square root of the test information function, $I^*(\tau)$, for the τ obtained by the transformation of θ to τ through the three polynomials given by (3.4) is shown by the three dotted

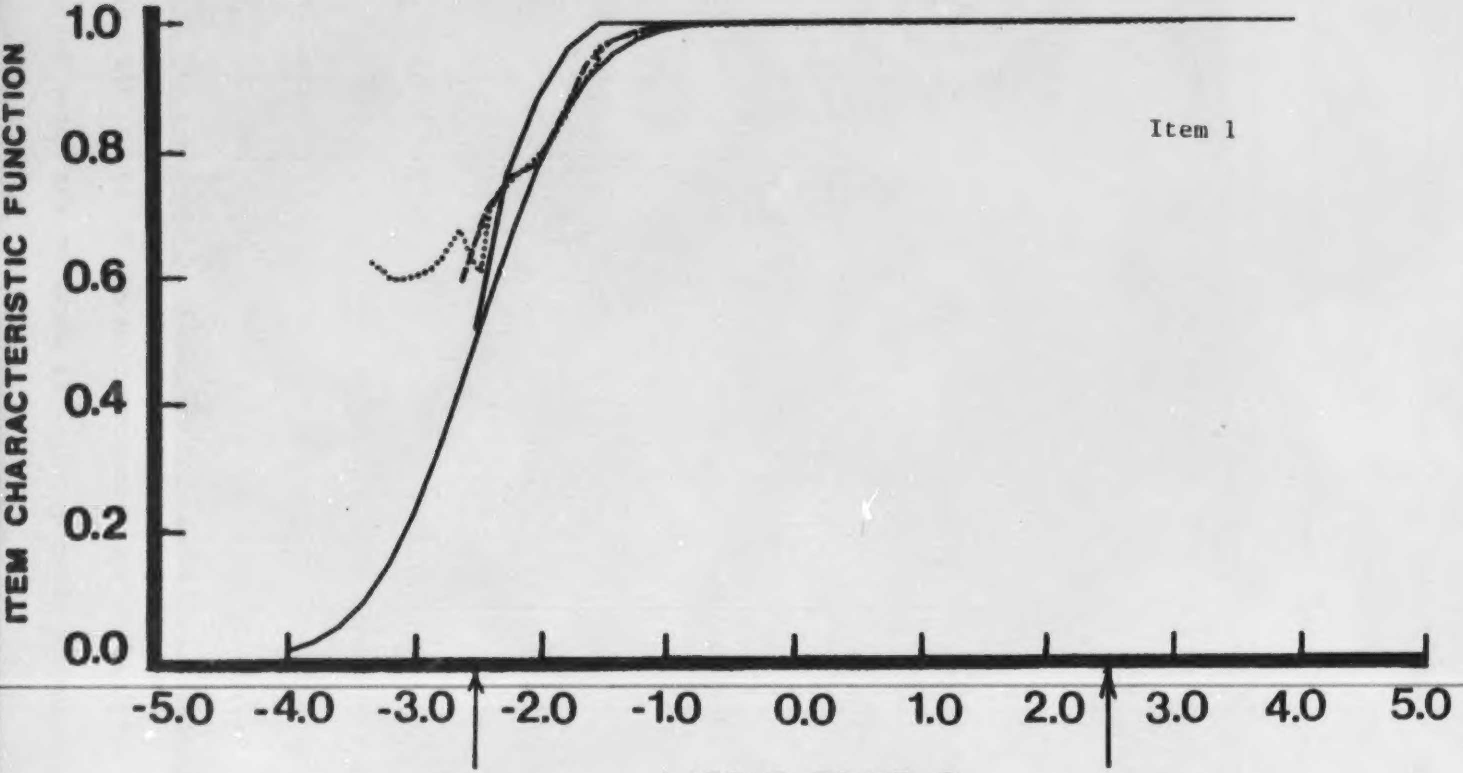


FIGURE 3-5

Estimated Item Characteristic Function (Dotted Curve) Based upon Subtest 2, but Using the Set of Three Different Transformations of $\hat{\theta}$ to $\hat{\tau}$.
Degree 3 Case

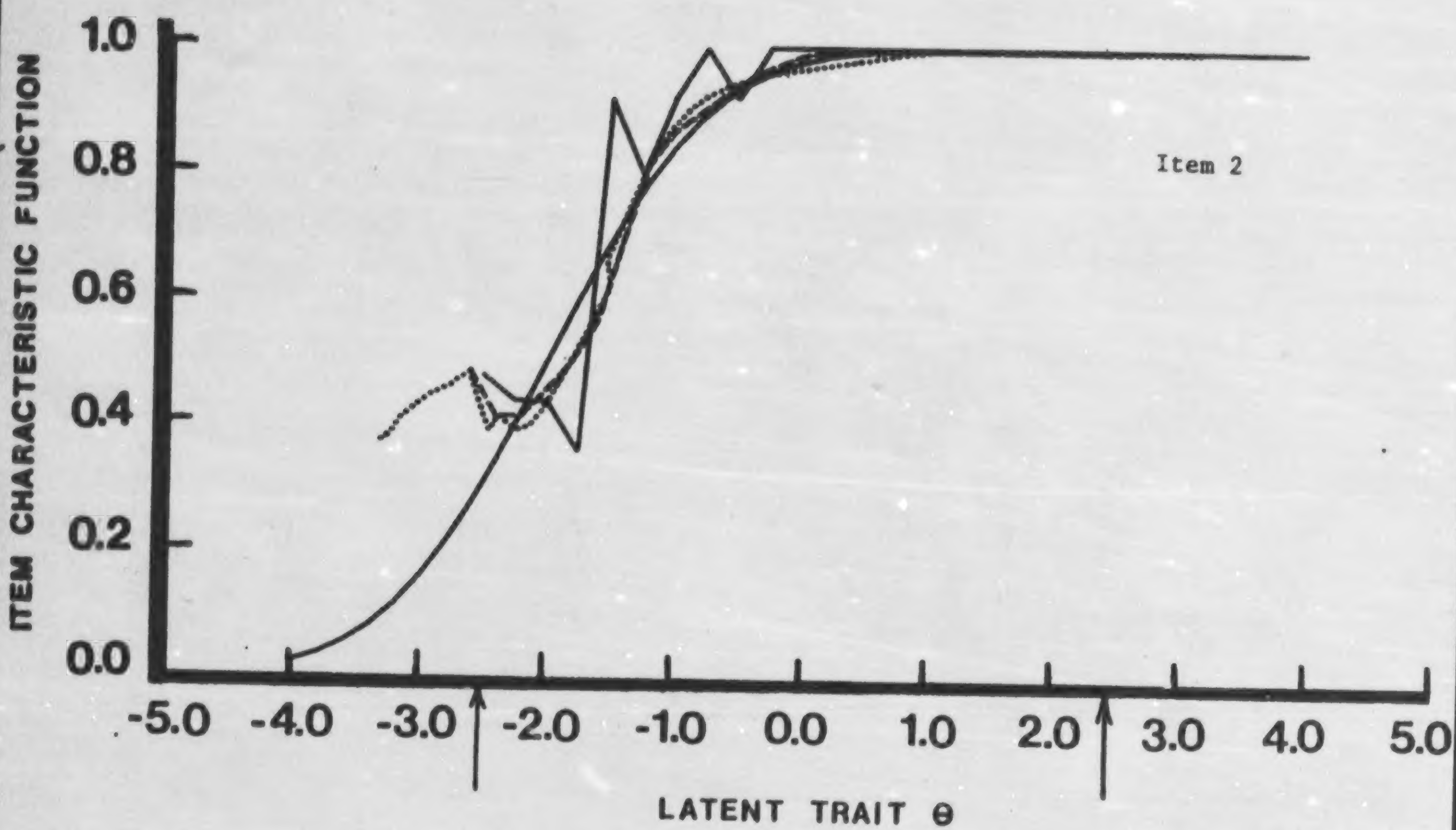


FIGURE 3-5 (Continued): Subtest 2 with Three Different Transformations, Degree 3 Case.

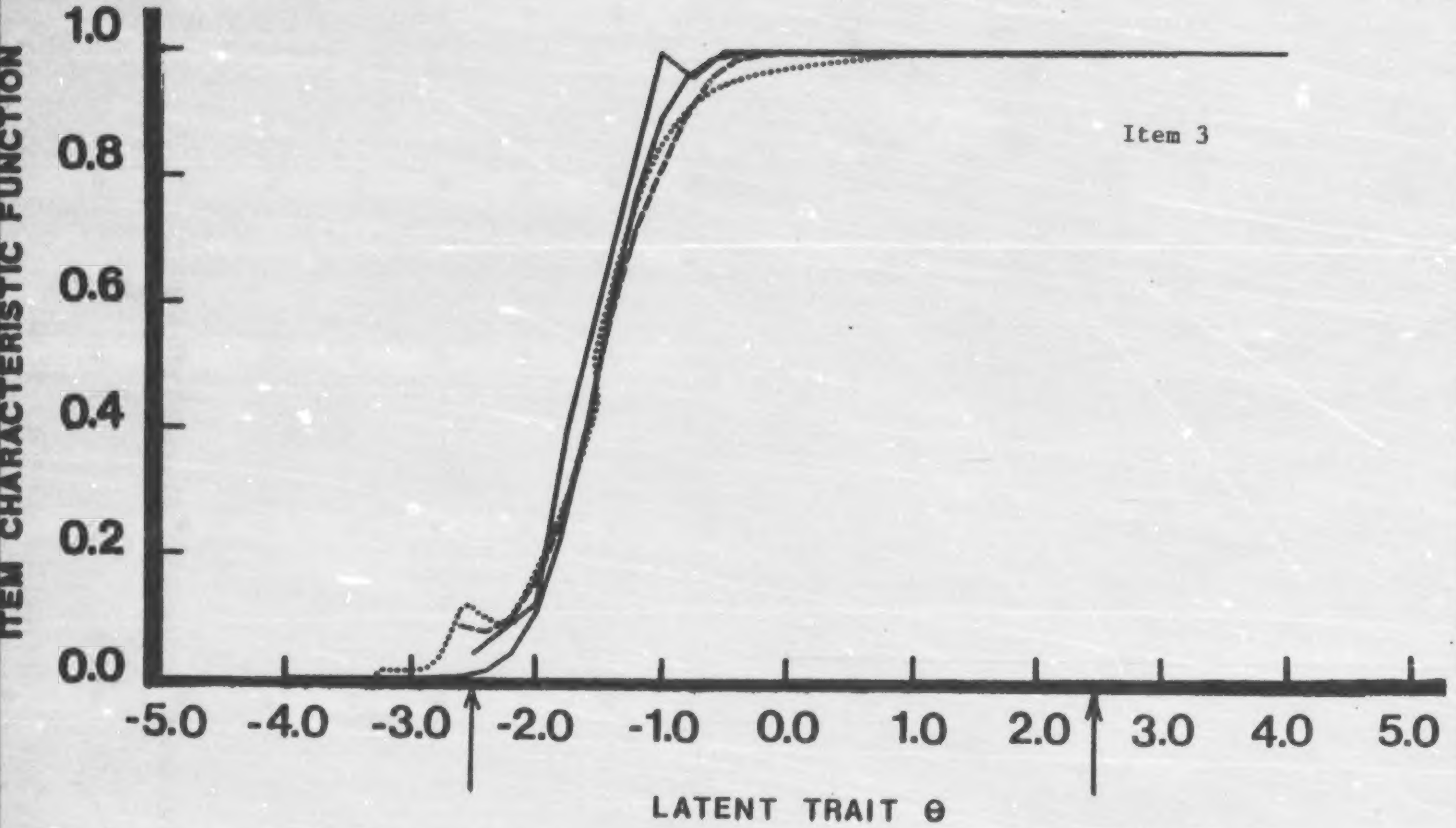


FIGURE 3-5 (Continued): Subtest 2 with Three Different Transformations, Degree 3 Case.

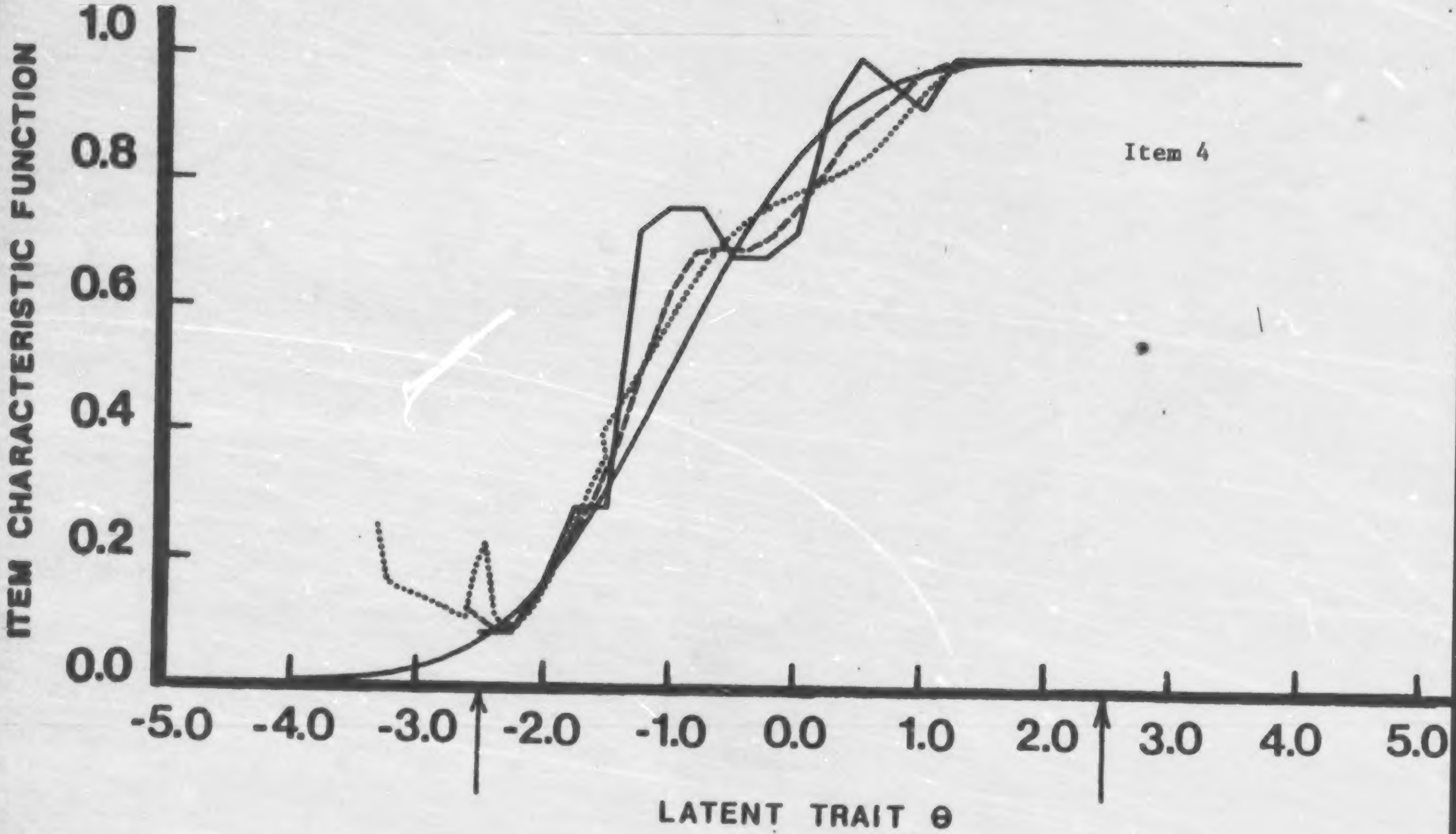


FIGURE 3-5 (Continued): Subtest 2 with Three Different Transformations, Degree 3 Case.

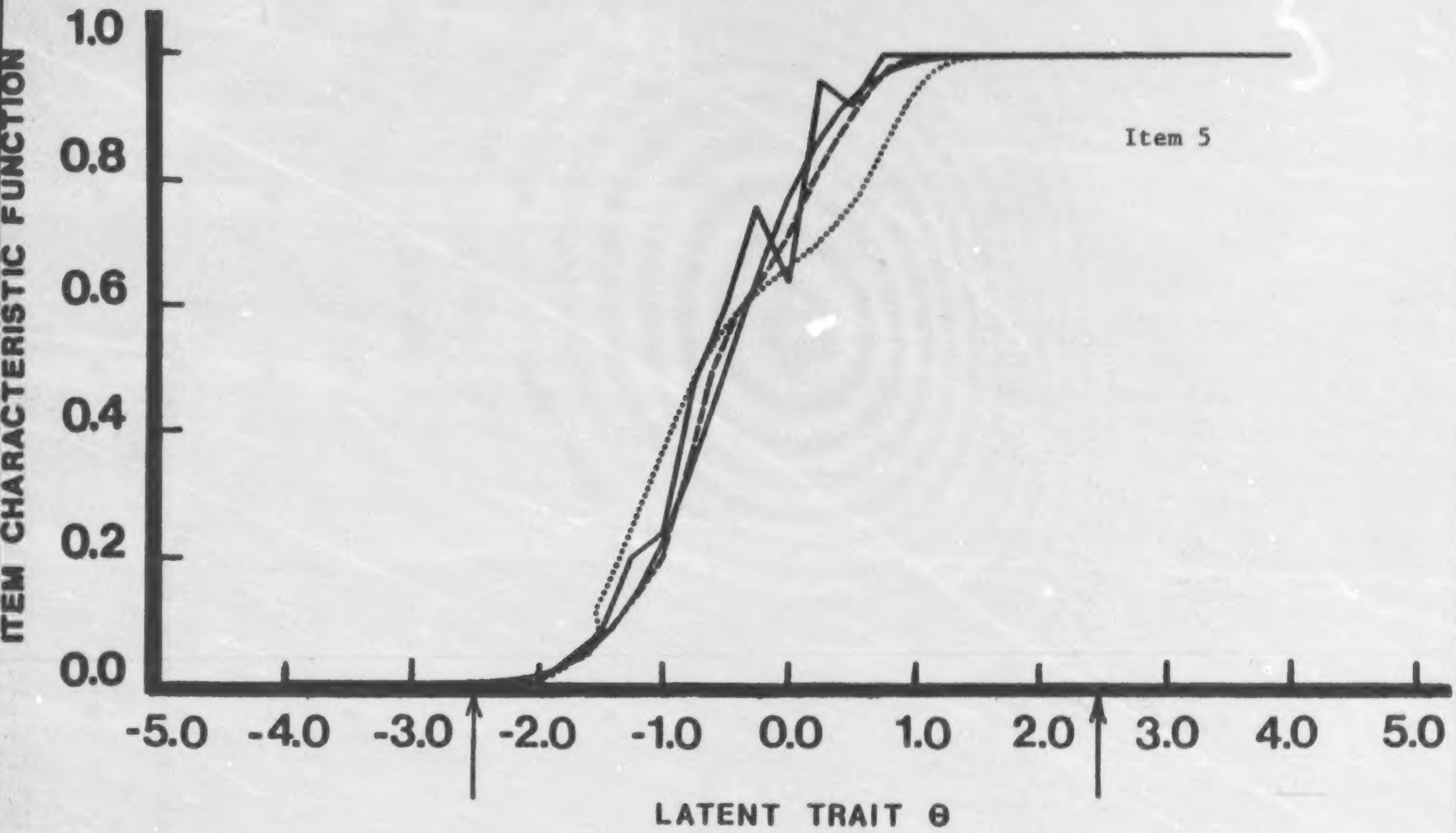


FIGURE 3-5 (Continued): Subtest 2 with Three Different Transformations, Degree 3 Case.

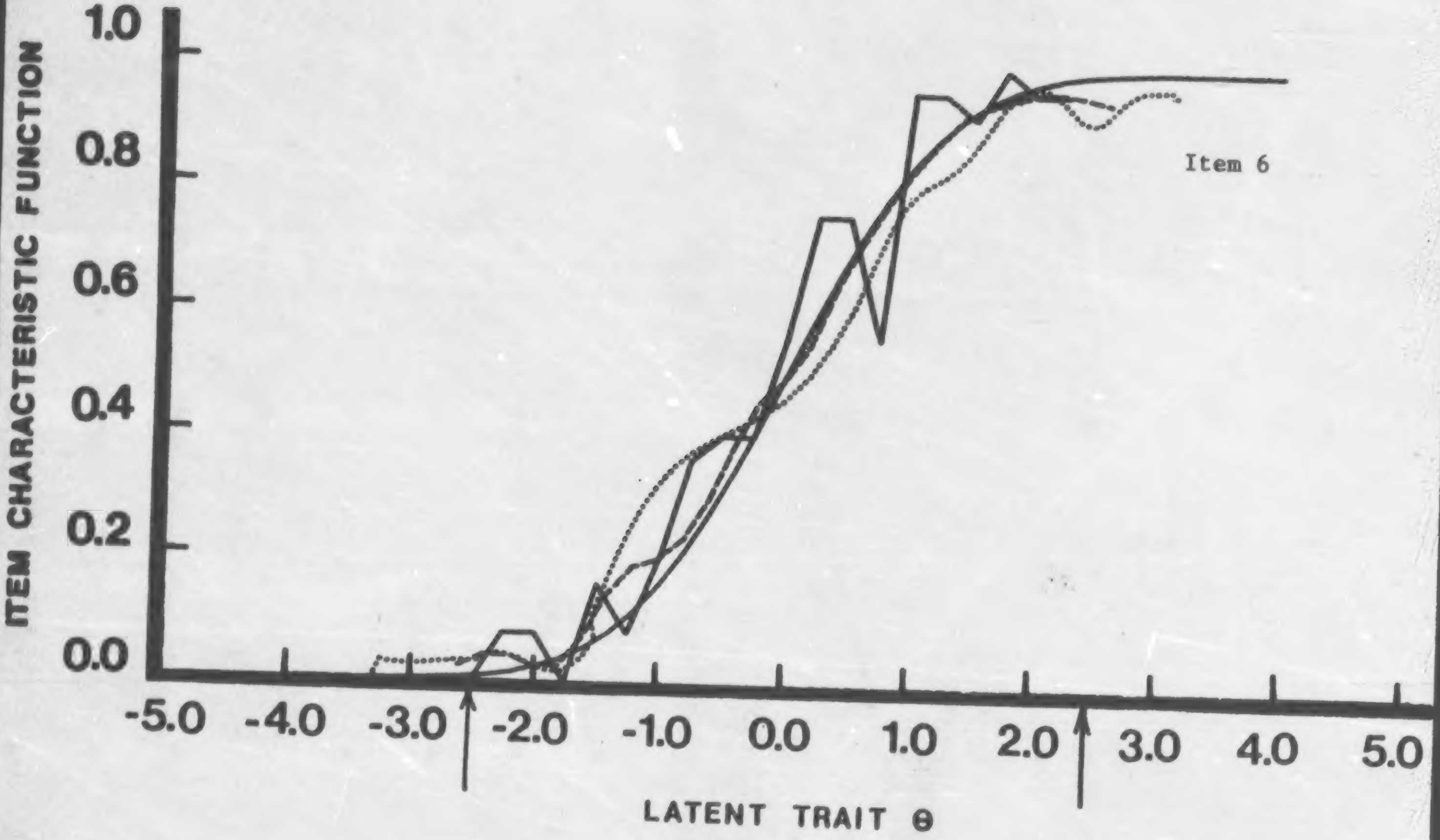


FIGURE 3-5 (Continued): Subtest 2 with Three Different Transformations, Degree 3 Case.

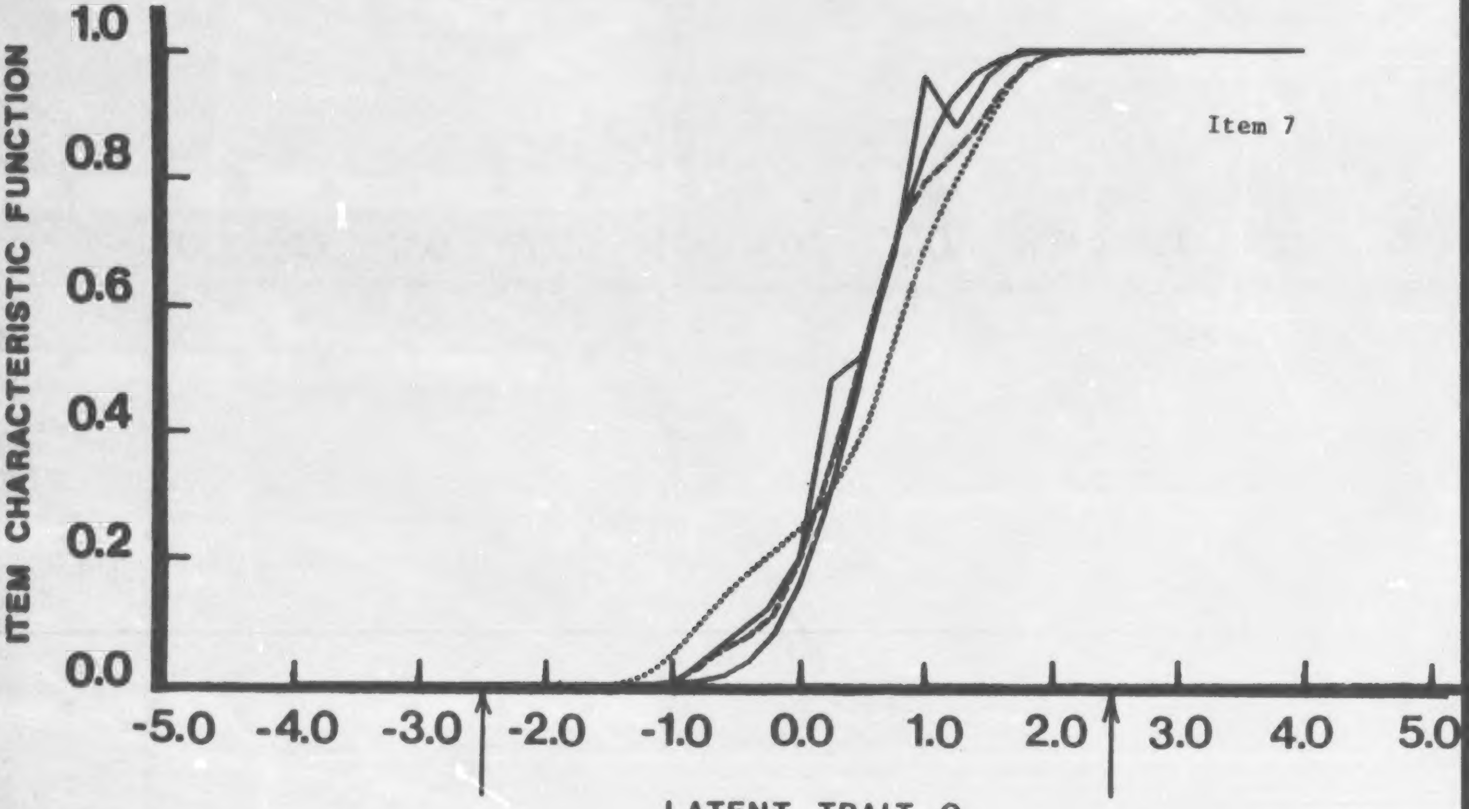


FIGURE 3-5 (Continued): Subtest 2 with Three Different Transformations, Degree 3 Case.

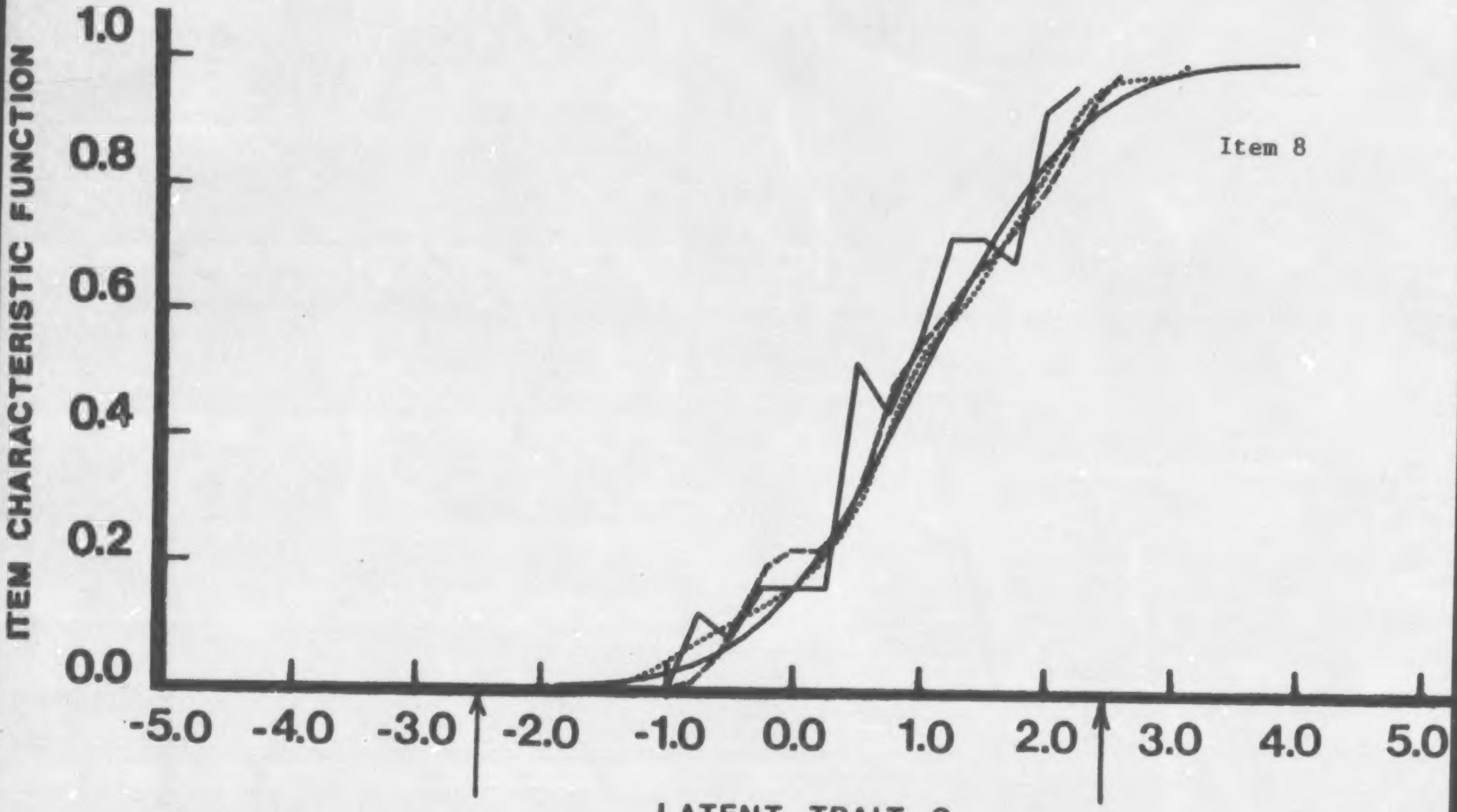


FIGURE 3-5 (Continued): Subtest 2 with Three Different Transformations, Degree 3 Case.

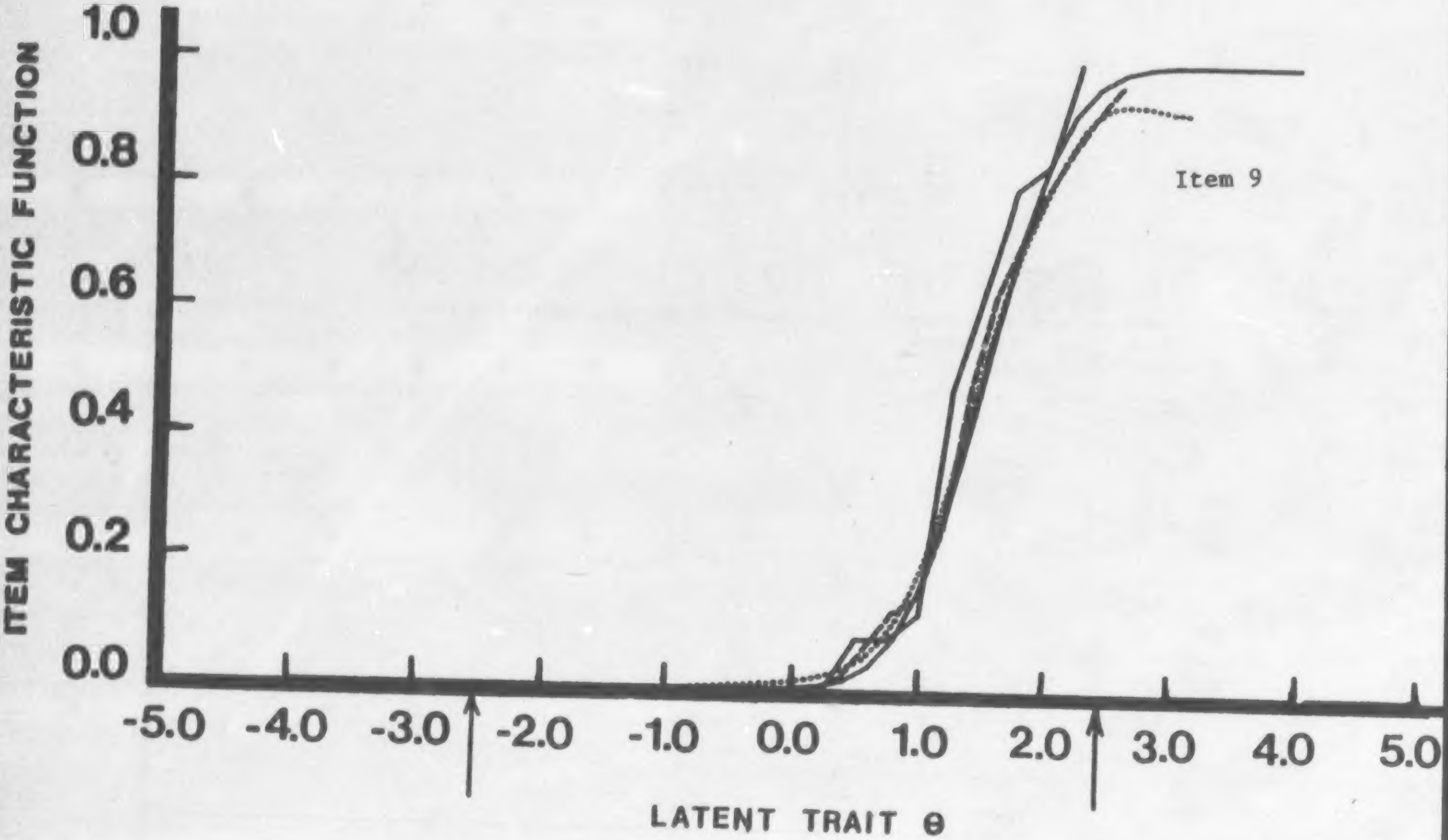


FIGURE 3-5 (Continued): Subtest 2 with Three Different Transformations, Degree 3 Case.

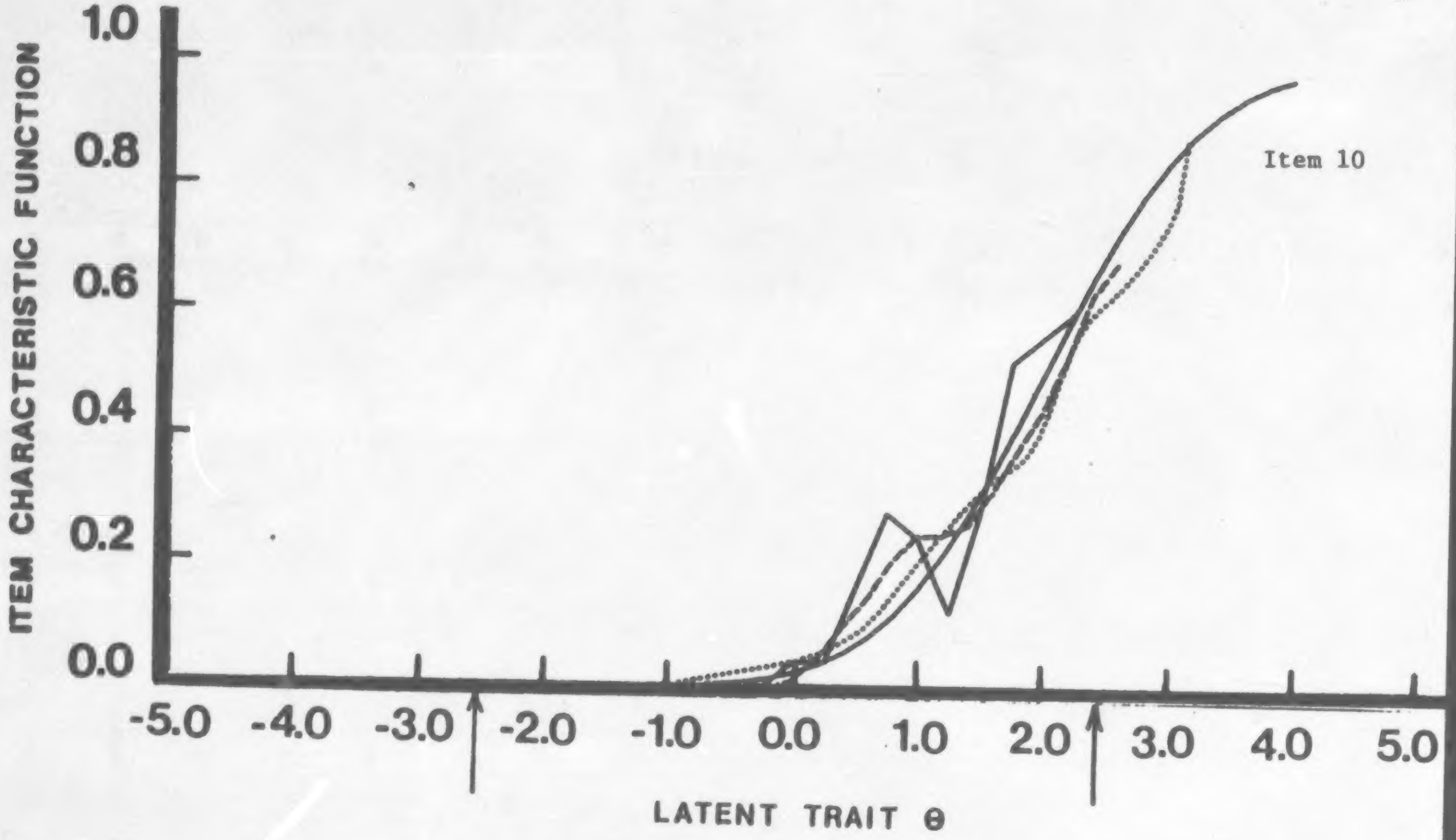
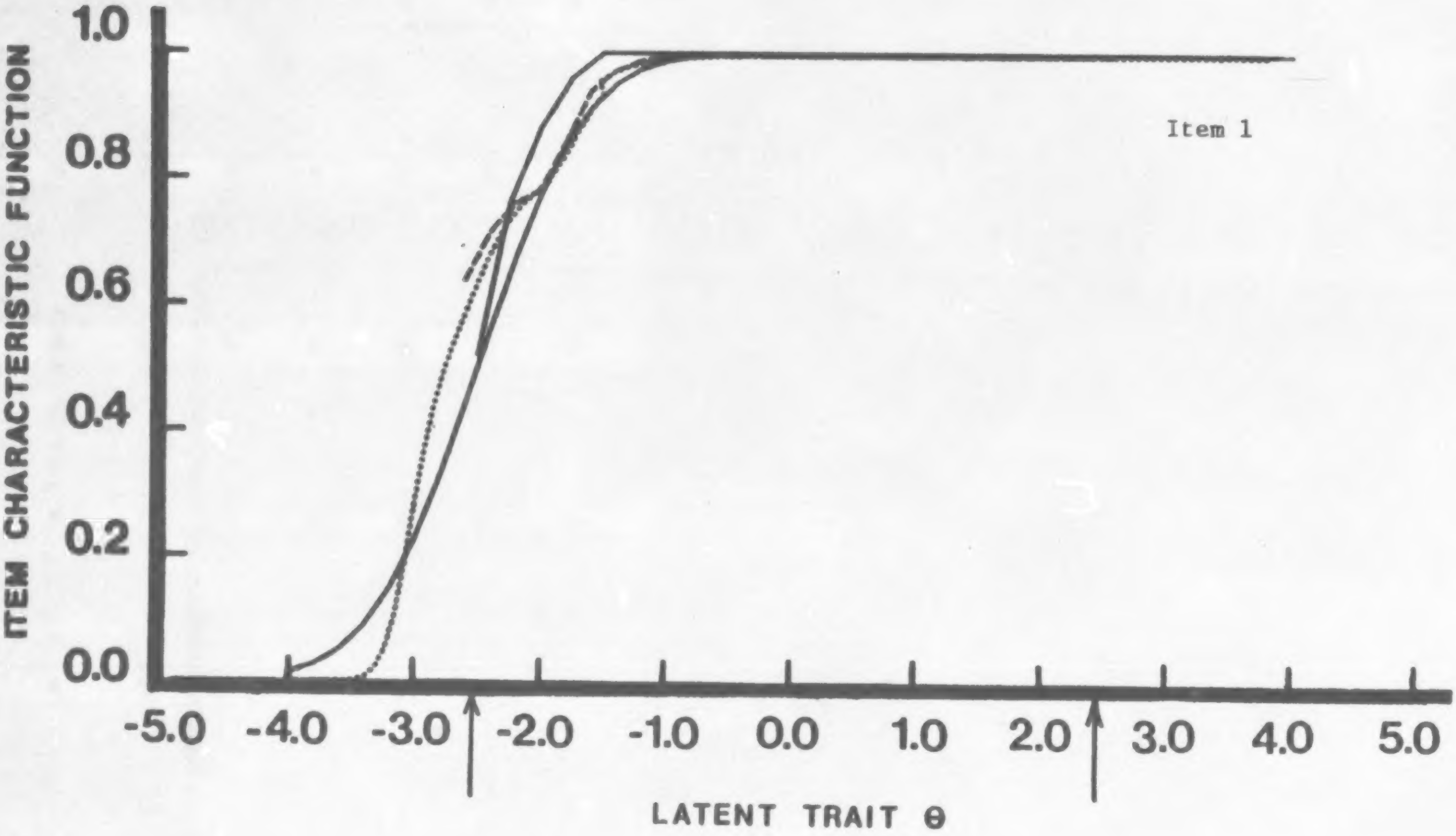


FIGURE 3-5 (Continued): Subtest 2 with Three Different Transformations, Degree 3 Case.



Estimated Item Characteristic Function (Dotted Curve) Based upon Subtest 2, but Using the Set of Three Different Transformations of $\hat{\theta}$ to $\hat{\tau}$.
Degree 4 Case

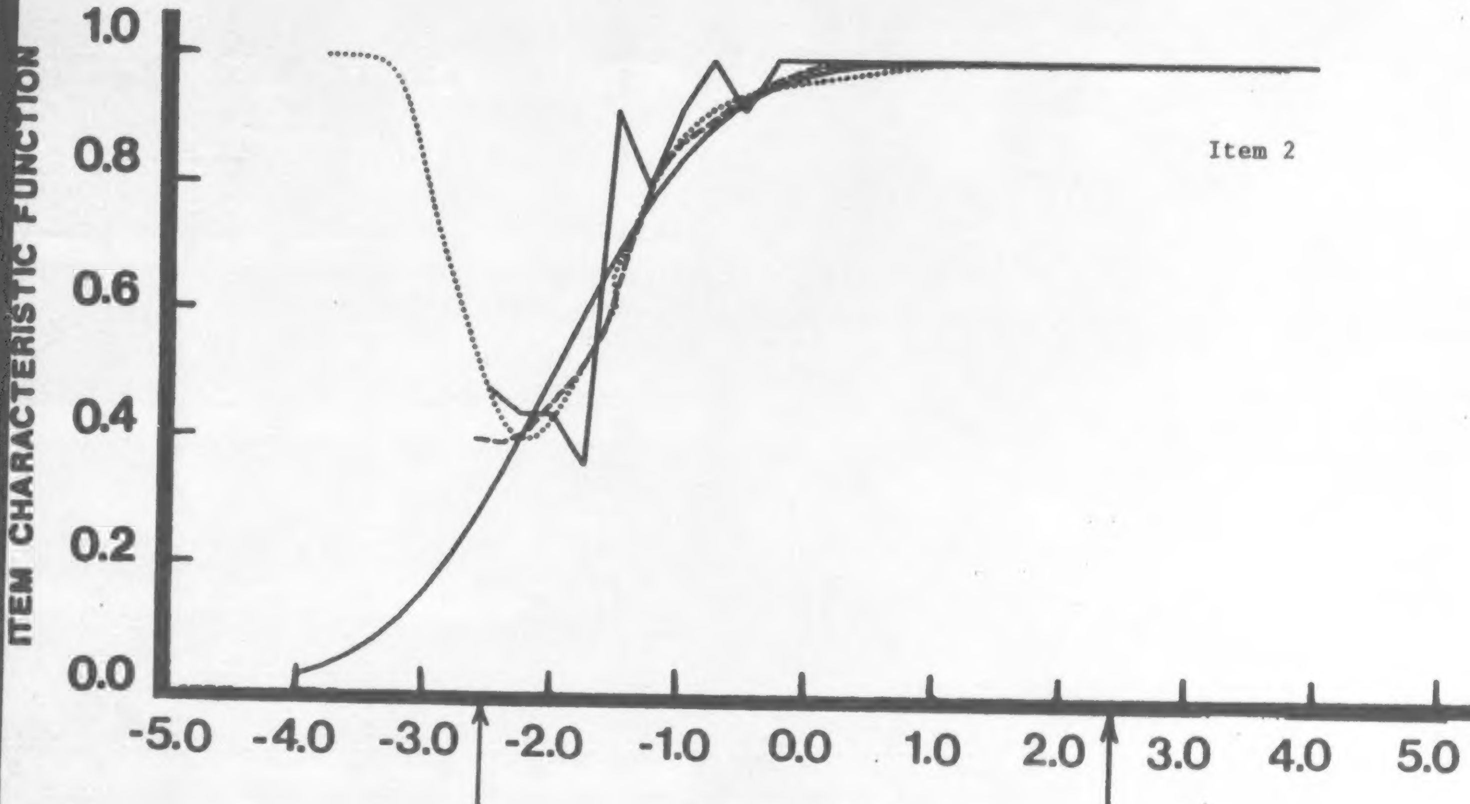


FIGURE 3-6 (Continued): Subtest 2 with Three Different Transformations, Degree 4 Case.

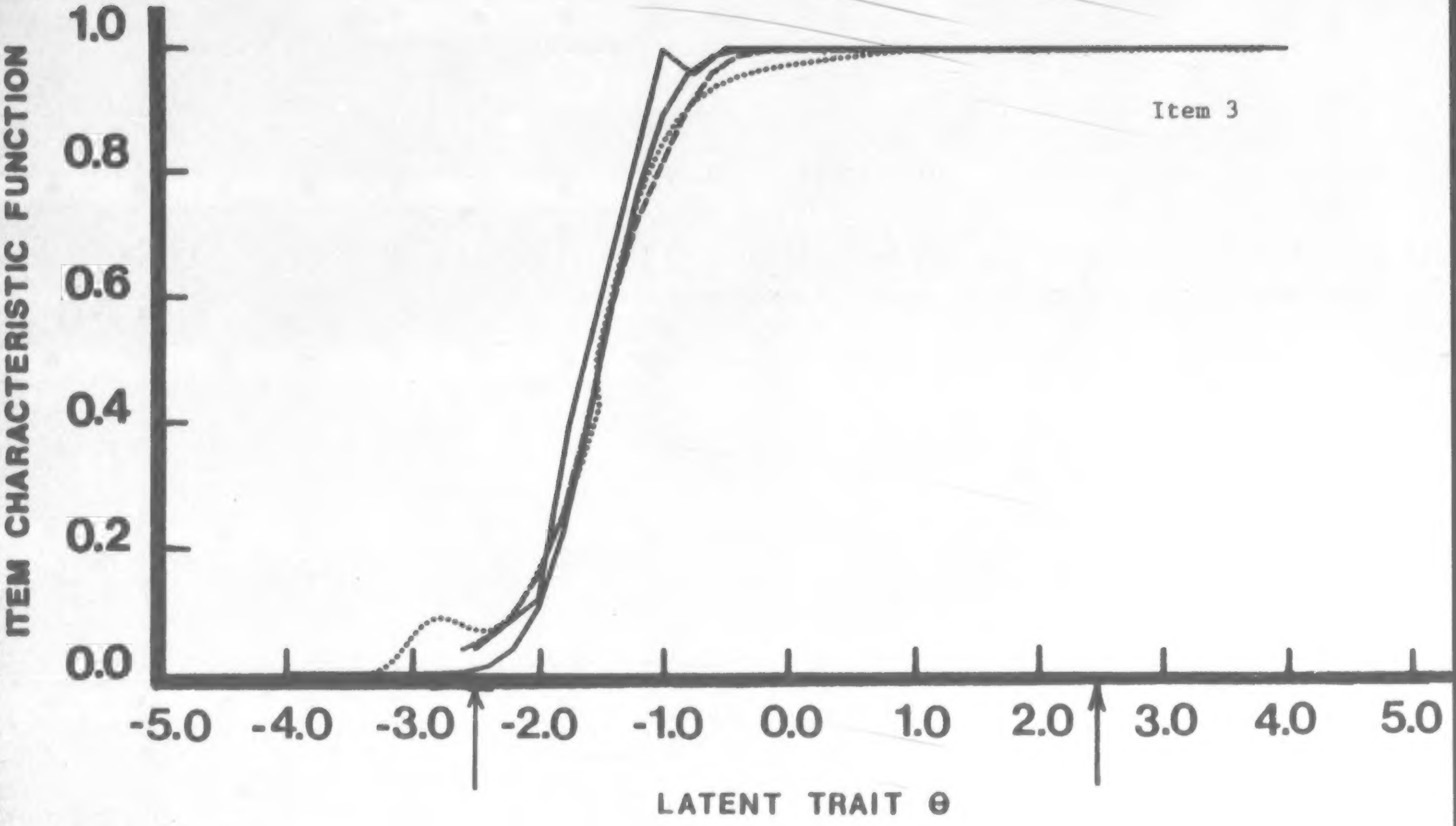


FIGURE 3-6 (Continued): Subtest 2 with Three Different Transformations, Degree 4 Case.

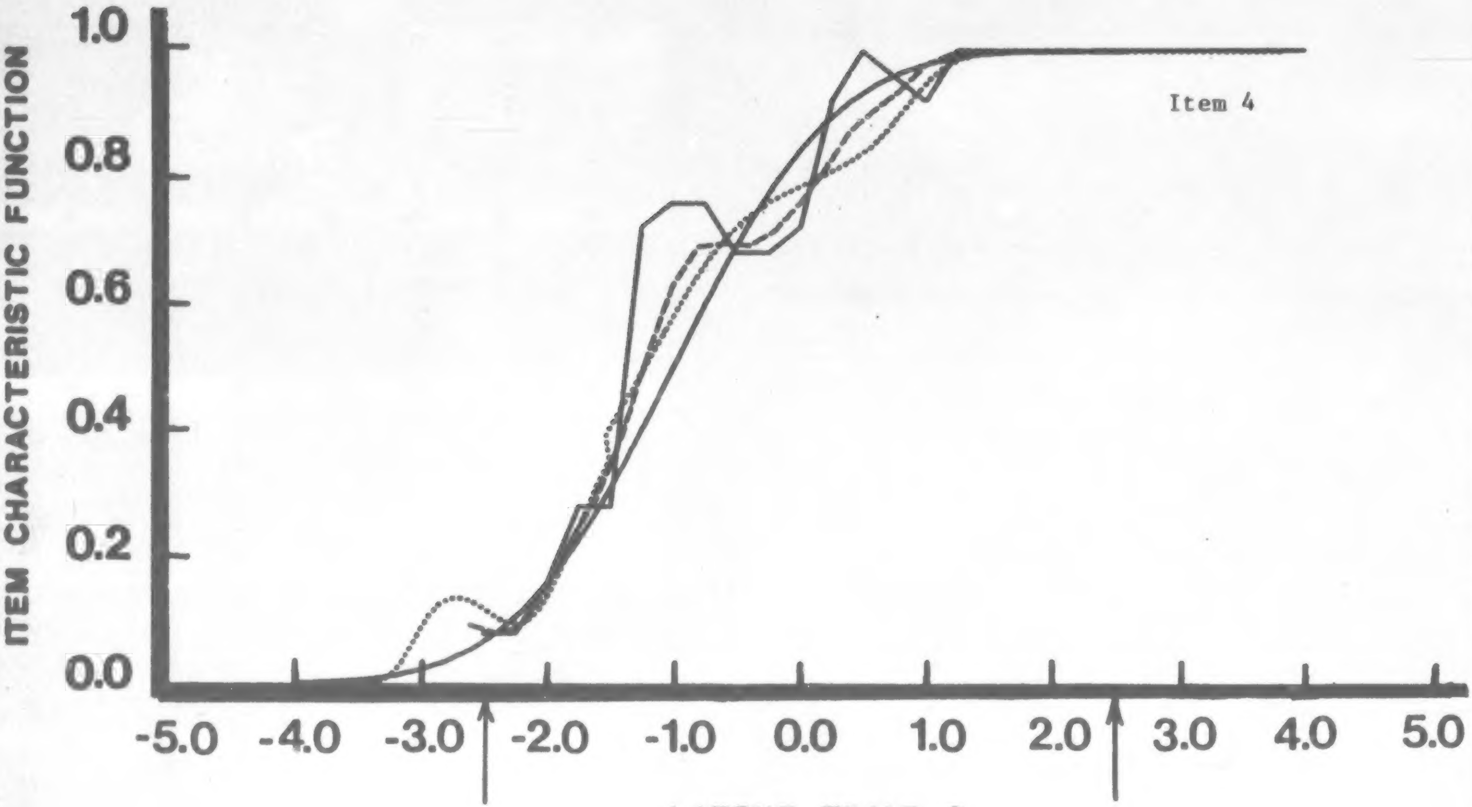


FIGURE 3-6 (Continued): Subtest 2 with Three Different Transformations, Degree 4 Case.

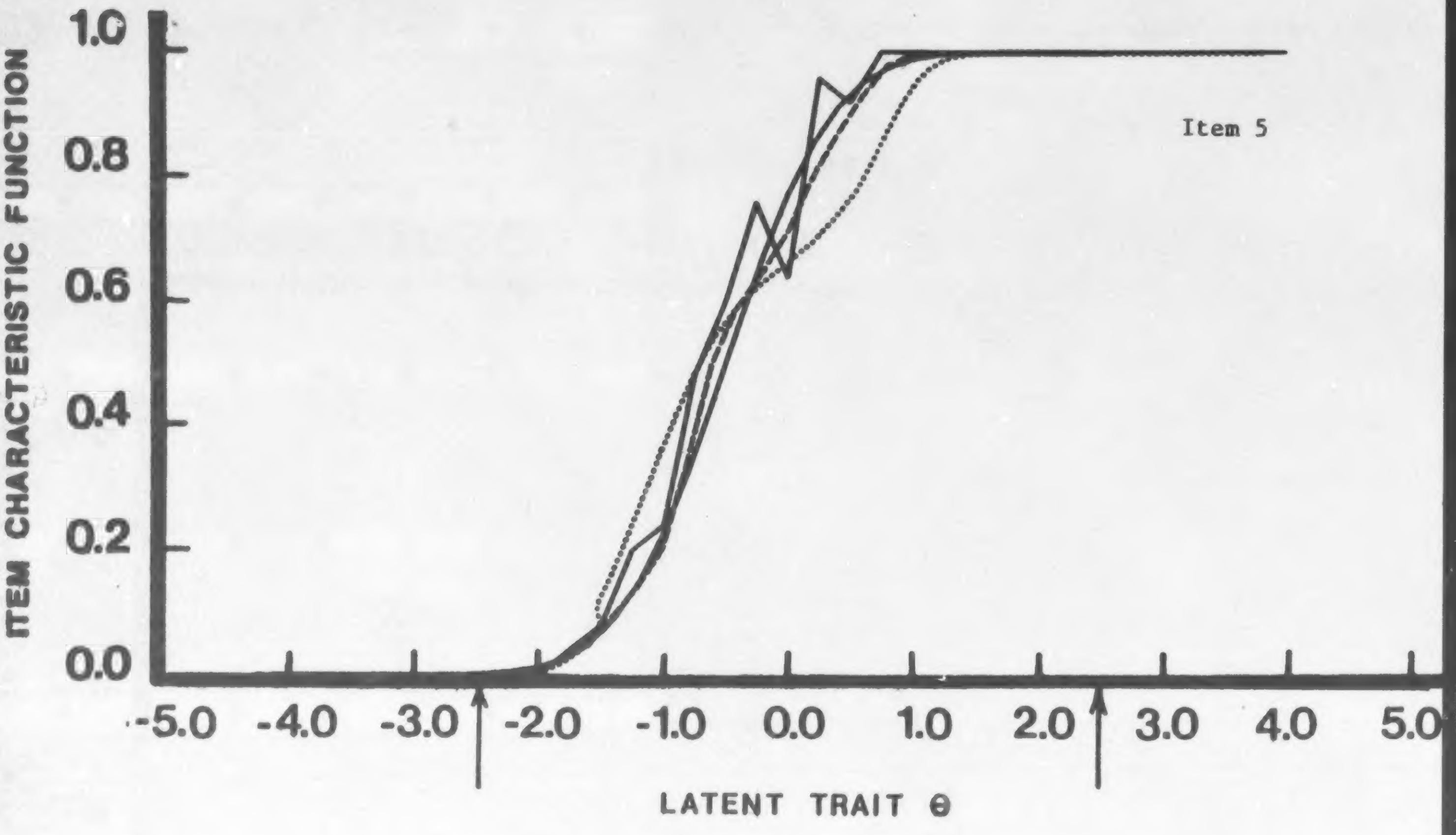


FIGURE 3-6 (Continued): Subtest 2 with Three Different Transformations, Degree 4 Case.

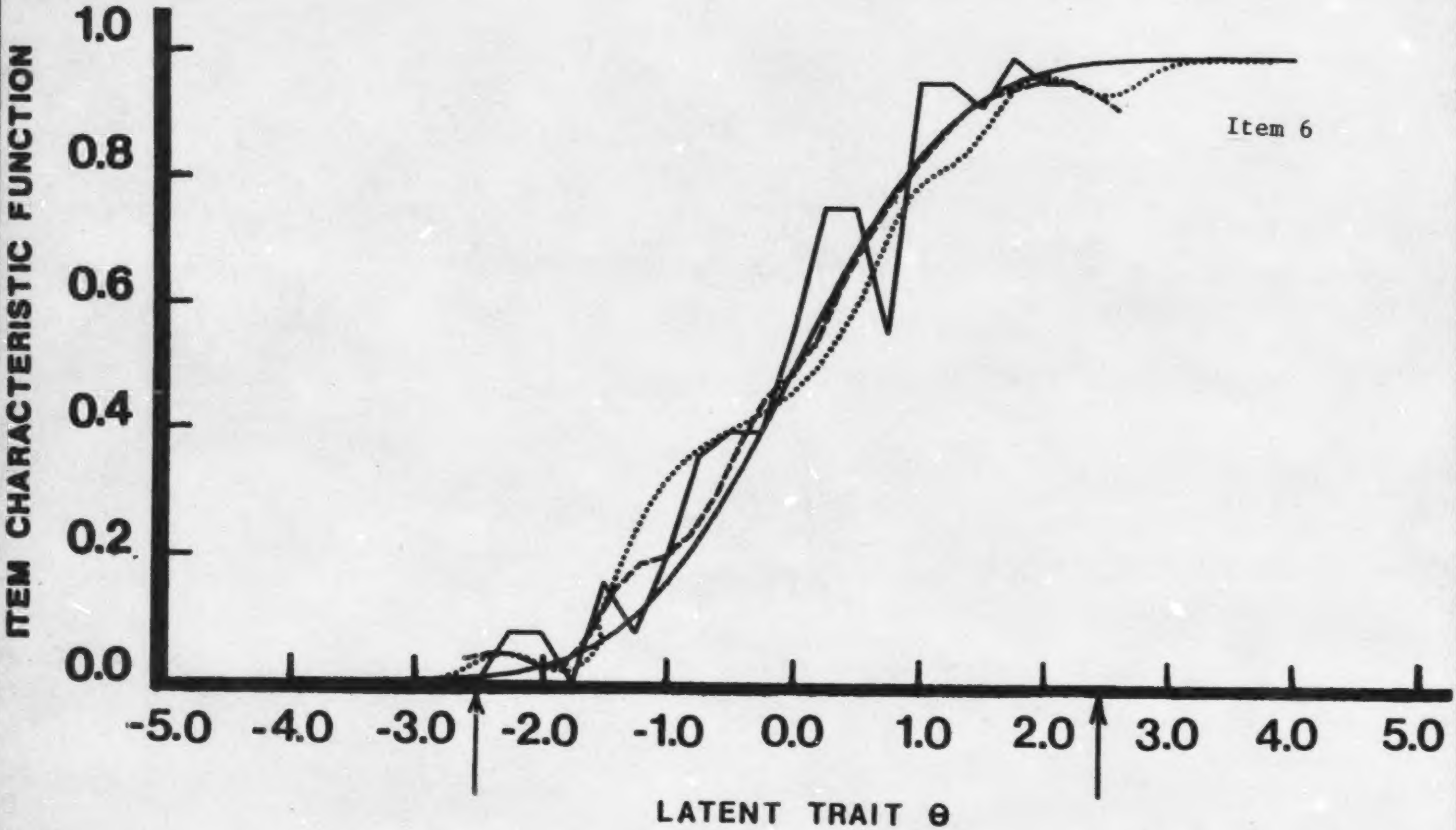


FIGURE 3-6 (Continued): Subtest 2 with Three Different Transformations, Degree 4 Case.

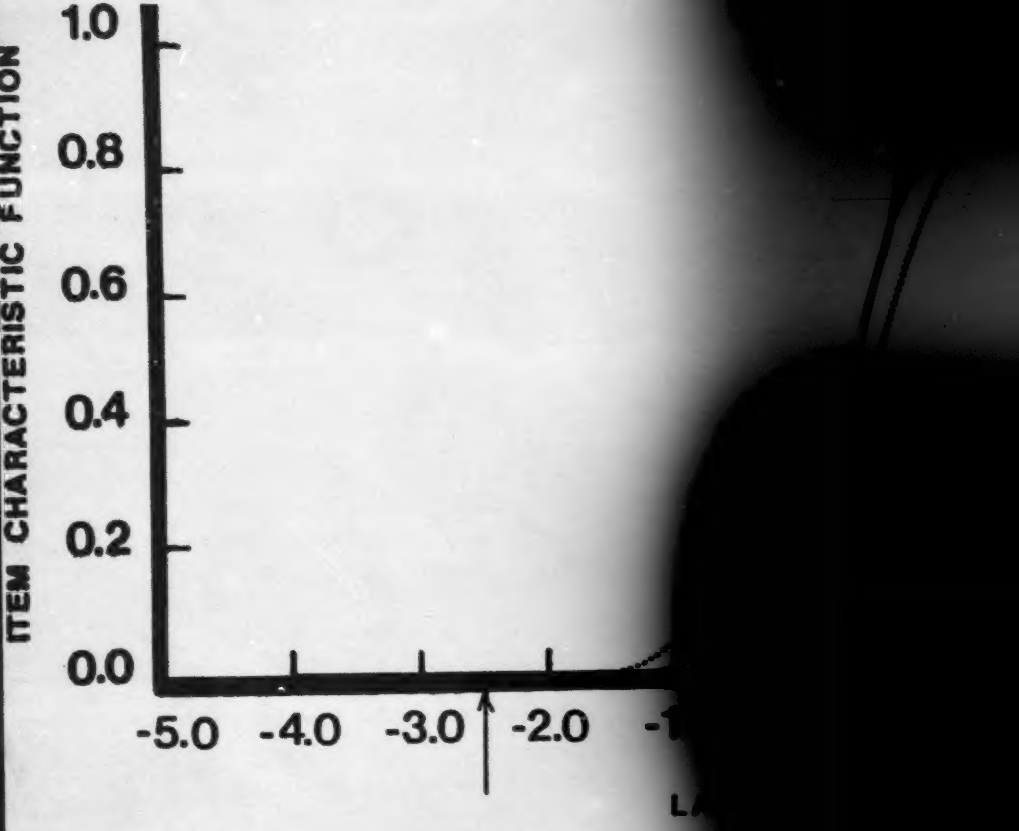


FIGURE 3-6 (Continued): Subtest 2 wi

curves in Figure 3-7, together with the constant function, C (= 3.5) . We can see that the result is much closer to C than the one obtained previously and drawn in Figure 3-2.

It should be noted that the two different values of C , 4.5 and 3.5 for Subtests 1 and 2, respectively, for the square root of the test information function after the transformation of θ to τ have nothing to do with the differences in the results obtained upon Subtests 1 and 2. These values were chosen more or less arbitrarily, in order to make the range of the five hundred τ_s's reasonably close to that of θ_s's . It is obvious from the formulas for the coefficients, α_k^* , in (2.8), however that a specific value for C does not affect the result.

The transformation of θ to τ is illustrated as Figure A-2-1 in Appendix II, for the two cases in which Subtests 1 and 2 were used in place of the original Old Test, respectively.

It is conceivable that the relative inaccuracy of estimation caused by using Subtest 2 in place of the original Old Test is due to the fact that the amount of test information around the middle of the interval of θ , (-2.5, 2.5) , in which all the ability levels of the five hundred hypothetical examinees are located, is far smaller than that of Subtest 1, as is obvious in Figures 2-1 and 2-2. The histogram of $\hat{\tau}_s$, which was obtained upon Subtest 2 and shown in Figure 2-4, discloses an extremely high frequency of $\hat{\tau}_s$ for the subinterval, (-0.5, 0.0) , and the same is true with the histogram shown in Figure 3-4, for the $\hat{\tau}_s$'s obtained upon Subtest 2

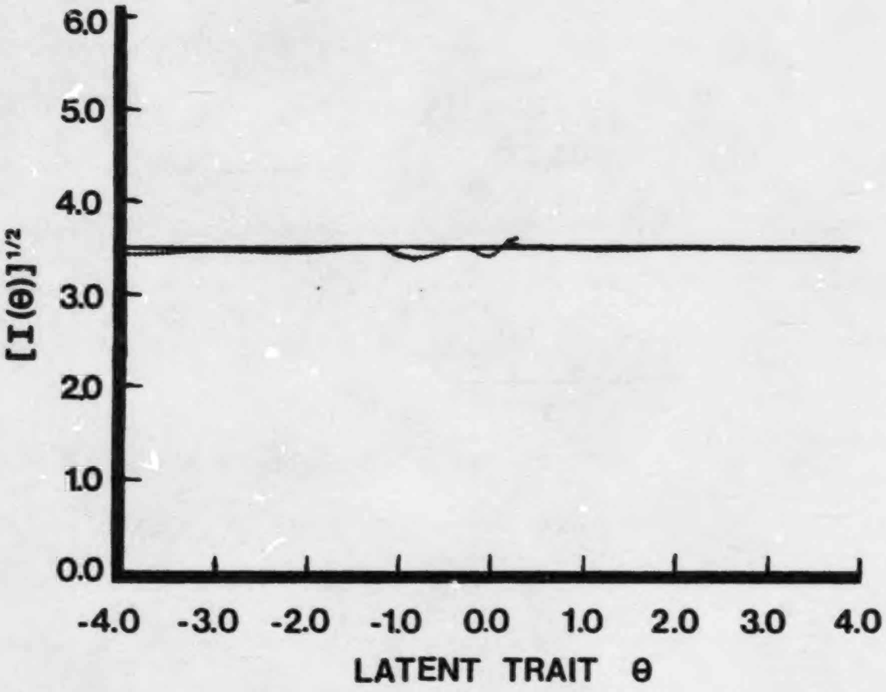


FIGURE 3-7

Square Root of the Constant Test Information, $[I^*(\theta)]^{1/2}$, of Subtest 2 (Dotted Curves) Using Three Polynomials in Transforming θ to τ, in Comparison with Its Theoretical Values (Solid Line).

but by using the set of three polynomials in transforming θ to τ. This is caused by the fact that as many as 98 examinees obtained an identical response pattern, (2,2,2,2,2,2,2,2,2,2,2,2,0,0,0,0,0,0,0,0,0,0,0,0,0) , i.e. the highest item scores for items 1 through 12 and the lowest item scores for items 23 through 35 of the original Old Test of which Subtest 2 consists. We note that the first twelve items of Subtest 2 are far easier than the second thirteen items, and this is the reason why the amount of test information is very small around the middle of the interval of θ , (-2.5, 2.5) . The maximum likelihood estimates, $\hat{\theta}$, for these 98 examinees are uniformly -0.1202 , and those after the transformation of θ to τ are -0.0731 .

Figures 3-8 and 3-9 present the scatter diagrams of the five hundred examinees with respect to their τ_s's and $\hat{\tau}_s$'s , the latter of which were obtained upon Subtests 1 and 2, respectively. We can see in these figures that these two scatter diagrams are substantially different from each other, especially around the middle of the configuration.

It should be recalled that the rationale behind the present method includes the assumption that the maximum likelihood estimate, $\hat{\tau}$, is approximately unbiased, and its conditional distribution given τ , is approximately normal, with C^{-1} as the standard deviation, i.e., the second parameter of the normality. For this reason, in each of Figures 3-8 and 3-9, a thick straight line is drawn with 45 degrees from the abscissa to indicate the unbiasedness,

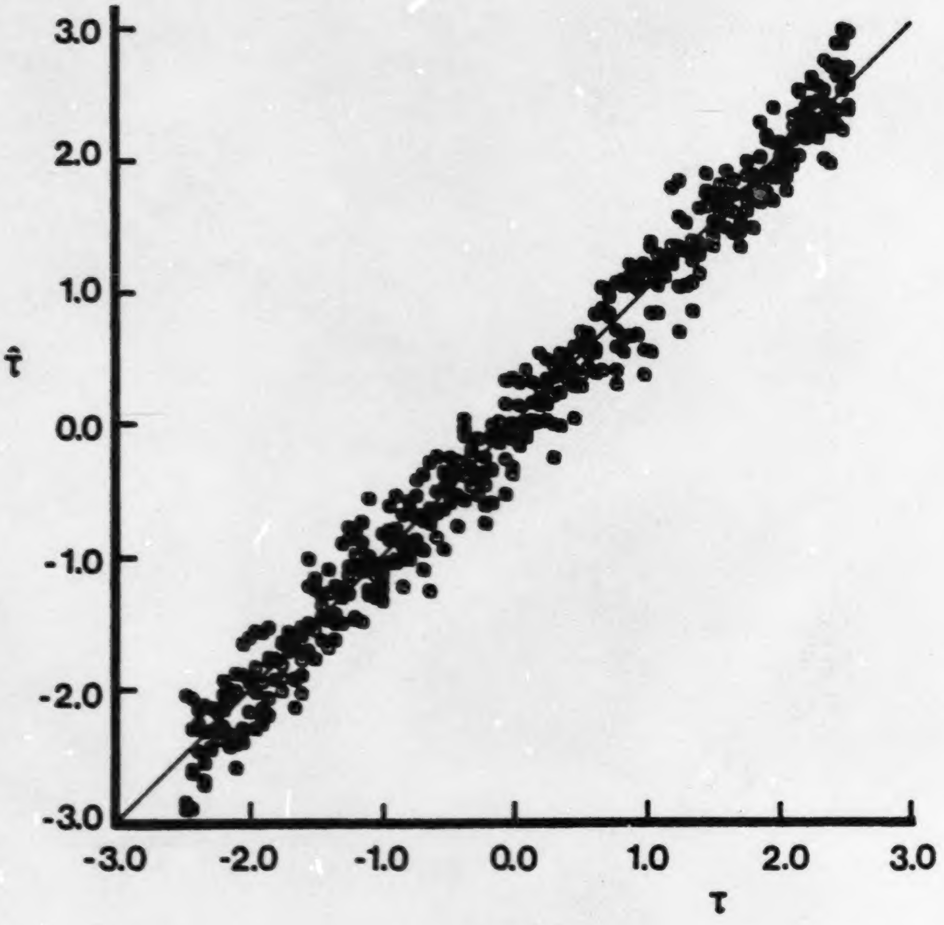


FIGURE 3-8

Maximum Likelihood Estimate, $\hat{\tau}$, Which Was Obtained Upon Subtest 1, Plotted Against τ , for Each of the Five Hundred Hypothetical Examinees.

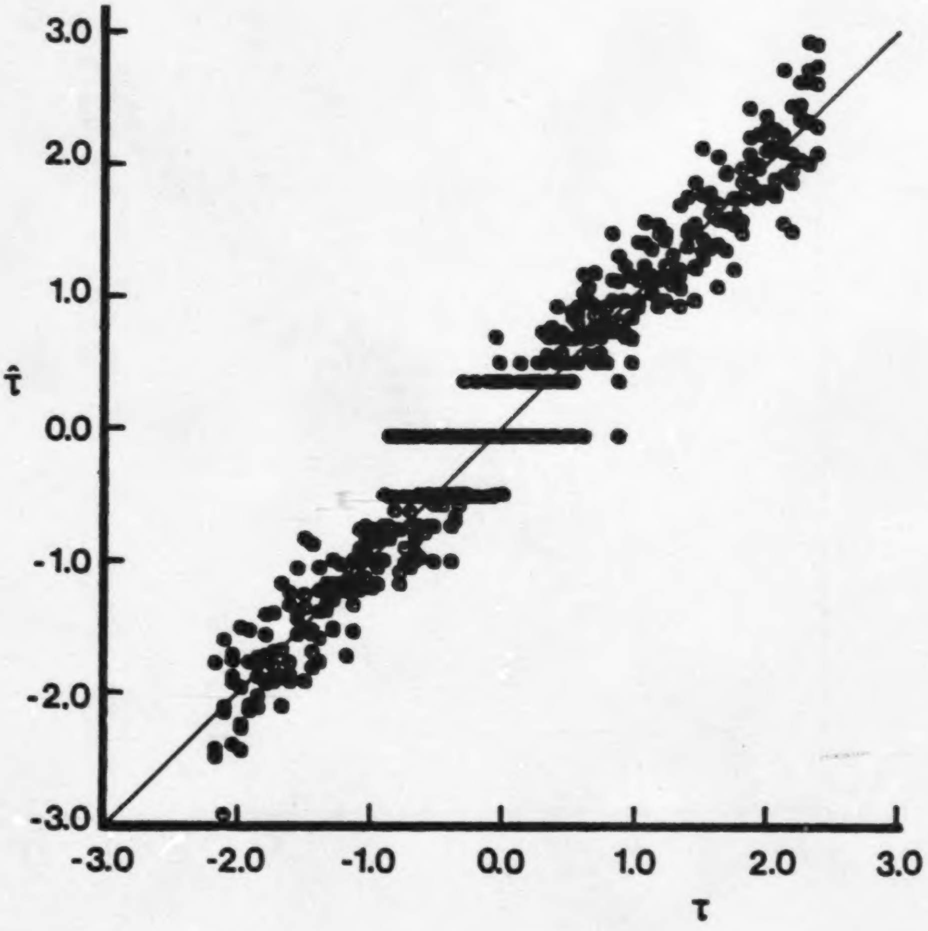


FIGURE 3-9

Maximum Likelihood Estimate, $\hat{\tau}$, Which Was Obtained Upon Subtest 2, Plotted Against τ , for Each of the Five Hundred Hypothetical Examinees.

and two thin lines are drawn to indicate the interval taken both above and below the thick line, with the distance of C^{-1}, i.e., approximately 0.222 for Subtest 1 and 0.286 for Subtest 2. With these lines as our aids, we can see that, while the scatter diagram based upon Subtest 1 suggests the satisfaction of the approximate unbiasedness and normality of the conditional distribution of $\hat{\tau}$, given τ, this is not the case with the result which is based upon Subtest 2, especially for the interval of τ, $(-1.0, 1.0)$. This was caused mainly by the 98 identical $\hat{\tau}_s$'s $(= -0.0731)$, which resulted from the identical response patterns for Subtest 2, as was mentioned earlier in the present chapter. It is obvious that the transformation of θ to τ in the effort of equalizing the amount of test information cannot ameliorate this fact.

IV Criterion Operating Characteristics

The criterion operating characteristic of a discrete item response for the Simple Sum Procedure of the Conditional P.D.F. Approach (Samejima, 1978a, 1978b) has been defined and used to provide us with a criterion that the given set of data can possibly attain by following the Simple Sum Procedure, without the error of estimation in approximating the conditional density of θ, given $\hat{\theta}$. Let $R_{x_g}(\theta)$ denote the criterion operating characteristic of the discrete item response x_g. We can write

$$R_{x_g}(\theta) = \sum_{s \varepsilon x_g} \gamma(\theta|\hat{\theta}_s)\Big[\sum_{s=1}^{N} \gamma(\theta|\hat{\theta}_s)\Big]^{-1} , \tag{4.1}$$

where $\hat{\theta}_s$ is the maximum likelihood estimate of ability θ for the individual examinee s, and $\gamma(\theta|\hat{\theta}_s)$ is the true conditional density of θ, given $\hat{\theta}_s$. We notice that, if we replace this true conditional density function, $\gamma(\theta|\hat{\theta}_s)$, by its estimate (4.1) will become the estimated operating characteristic of the discrete item response x_g, which is obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach. It is obvious, therefore, that the criterion operating characteristic is not observable with actual data, but is observable only with simulated data. When x_g is an binary item score, the criterion operating characteristic for $x_g = 1$ is called the criterion item characteristic function. It has been shown (Samejima, 1978a, 1978b) that, with our present data, the estimated item characteristic functions for the ten binary items obtained by the Simple Sum Procedure in combination with the Normal Approach Method, the Pearson System Method or the Two-Parameter Beta Method are practically identical with the criterion item characteristic functions, provided that we use the whole set of thirty-five items as our Old Test. The curves drawn by dashes is Figures 2-5, 2-6, 2-7, 2-8, 3-5, and 3-6 are, therefore, practically the best possible estimates of the item characteristic functions that the Procedure can produce, with the set of five hundred maximum likelihood estimates $\hat{\theta}_s$, obtained upon the original Old Test, and the given set of five hundred response patterns for the ten binary items.

This conditional density function, $\gamma(\theta|\hat{\theta}_s)$, is given by

$$\gamma(\theta|\hat{\theta}_s) = \zeta(\hat{\theta}_s|\theta)\ f(\theta)\ [h(\hat{\theta}_s)]^{-1} \quad , \tag{4.2}$$

where $\zeta(\hat{\theta}_s|\theta)$ is the conditional density of the maximum likelihood estimate $\hat{\theta}_s$ for the individual examinee s, given θ, $f(\theta)$ is the density function of θ, and $h(\hat{\theta}_s)$ is the density function of the maximum likelihood estimate $\hat{\theta}$ at $\hat{\theta} = \hat{\theta}_s$, for which we can write

$$h(\hat{\theta}_s) = \int_{-\infty}^{\infty} \zeta(\hat{\theta}_s|\theta)\ f(\theta)\ d\theta \quad . \tag{4.3}$$

It can easily be shown that this criterion operating characteristic has a transformation-free character. Let

τ be any strictly increasing function of θ , and $R^*_{x_g}(\tau)$ be the criterion operating characteristic of the discrete item response x_g defined for the transformed latent trait τ . Since there is a one-to-one correspondence between the two maximum likelihood estimates, $\hat{\theta}$ and $\hat{\tau}$, we can write for the conditional density function, $\phi(\tau|\hat{\tau}_s)$, of τ , given $\hat{\tau}_s$,

$$(4.4) \qquad \phi(\tau|\hat{\tau}_s) = \gamma(\theta|\hat{\theta}_s) \frac{d\theta}{d\tau} \quad .$$

We obtain, therefore,

$$(4.5) \qquad R^*_{x_g}(\tau) = \sum_{s\varepsilon x_g} \phi(\tau|\hat{\tau}_s) \, [\sum_{s=1}^{N} \phi(\tau|\hat{\tau}_s)]^{-1}$$

$$= R_{x_g}(\theta) \quad .$$

When the test information function, $I(\theta)$, of the Old Test is not constant, it is easier to use the transformed latent trait, τ , for which the test information function, $I^*(\tau)$, is constant, and obtain the criterion operating characteristic, $R_{x_g}(\theta)$, through (4.5). We can write

$$(4.6) \qquad \phi(\tau|\hat{\tau}_s) = \psi(\hat{\tau}_s|\tau) \, f^*(\tau) \, [g(\hat{\tau}_s)]^{-1} \quad ,$$

where $\psi(\hat{\tau}_s|\tau)$ is the conditional density of the maximum likelihood estimate, $\hat{\tau}_s$, given τ , $f^*(\tau)$ is the density function of τ , and $g(\hat{\tau}_s)$ is the density function of $\hat{\tau}$ for $\hat{\tau} = \hat{\tau}_s$, which is given by

$$(4.7)\qquad g(\hat{t}_s) = \int_{-\infty}^{\infty} \psi(\hat{t}_s|\tau)\, f^*(\tau)\, d\tau \quad .$$

Since we have the relationship such that

$$(4.8)\qquad f^*(\tau) = f(\theta)\,\frac{d\theta}{d\tau} \quad ,$$

we can rewrite (4.6) and (4.7) in the forms

$$(4.9)\qquad \phi(\tau|\hat{t}_s) = \psi(\hat{t}_s|\tau)\, f(\theta)\,\frac{d\theta}{d\tau}\,[g(\hat{t}_s)]^{-1}$$

and

$$(4.10)\qquad g(\hat{t}_s) = \int_{-\infty}^{\infty} \psi(\hat{t}_s|\tau)\, f(\theta)\, d\theta \quad ,$$

respectively. Thus the true conditional density, $\phi(\tau|\hat{t}_s)$, is obtainable, by approximating the conditional density, $\psi(\hat{t}_s|\tau)$, by the normal density function, $n(\tau,C^{-1})$, where C is the square root of the constant test information, $I^*(\tau)$.

Using the uniform density function for the interval of θ , (-2.5, 2.5), as the theoretical density function, $f(\theta)$, the criterion item characteristic function was obtained from (4.8), (4.9) and (4.5), for each of the ten binary items, and for each of the two subsets.

Figure 4-1 presents the resultant ten criterion item characteristic functions obtained upon Subtest 1 by dotted curves, together with the corresponding criterion item characteristic functions obtained upon the original Old Test shown by dashed curves, and the theoretical item characteristic functions shown by solid curves. It is noted that, except for the very small discrepancies near the lower end of the interval of θ , (-2.5, 2.5), for items 1, 2, 3 and 4, and those near the upper end of the interval for item 6, these ten criterion item characteristic functions obtained upon Subtest 1 are practically identical with the corresponding functions obtained upon the original Old Test. This result was expected from the fact that the estimated item characteristic functions obtained by the Normal Approach Method upon Subtest 1 for the ten binary items proved to be practically identical with the corresponding estimates obtained upon the original Old Test, as we have observed in Chapter 2, and the fact that the latter proved to be practically identical with the corresponding criterion item characteristic functions (Samejima, 1978b).

Figure 4-2 presents the corresponding criterion item characteristic functions obtained upon Subtest 2, together with the other two sets of functions. Unlike the result obtained upon Subtest 1, from these ten graphs of Figure 4-2 we can say that the resultant criterion item characteristic functions are substantially different from those obtained upon the original Old Test. The differences are especially conspicuous for items 4, 5, 6 and 7, and

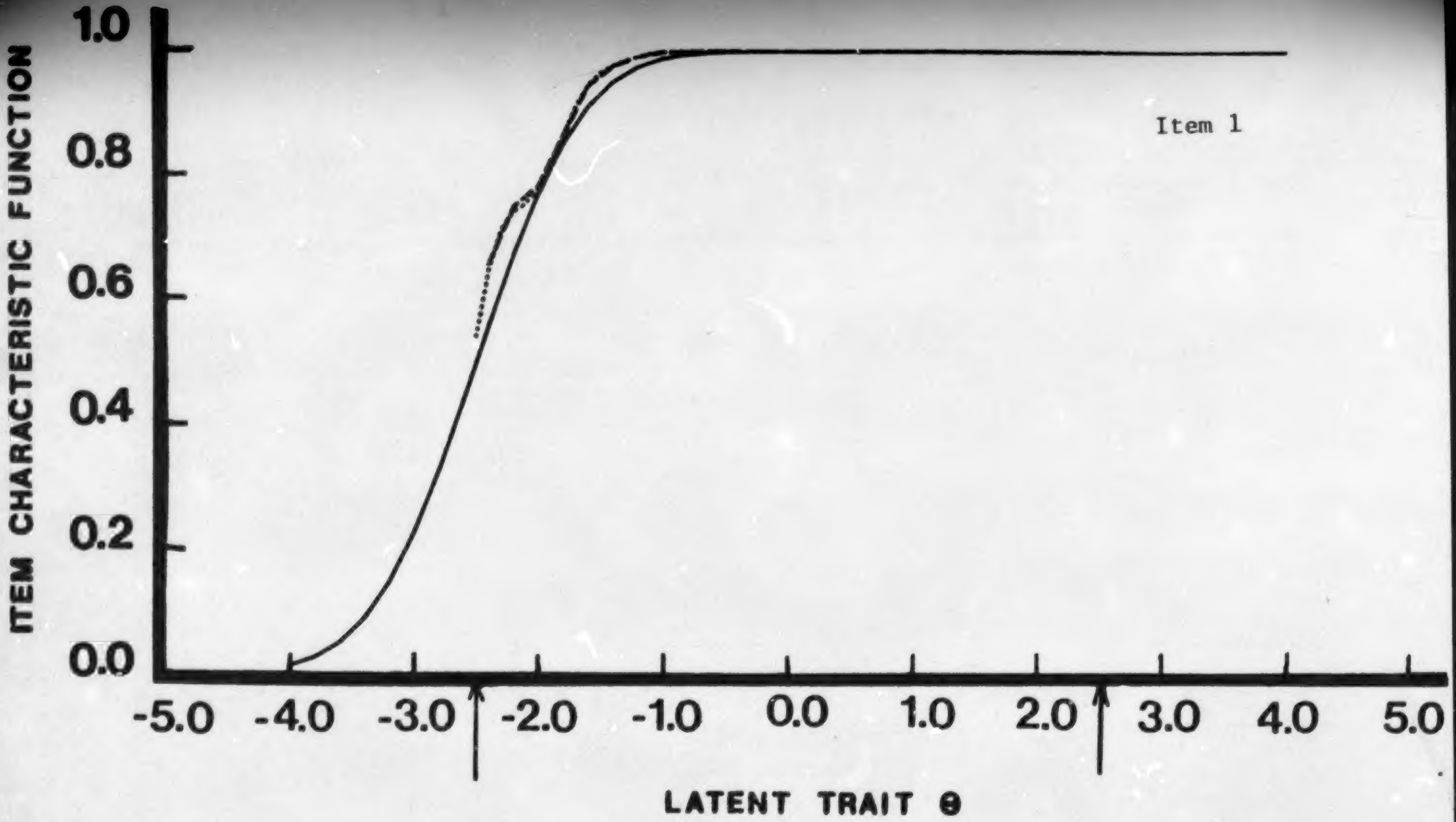


FIGURE 4-1

Criterion Item Characteristic Function Based Upon Subtest 1 (Dotted Curve), In Comparison With the One Obtained Upon the Original Old Test (Dashed Curve) And The Theoretical Item Characteristic Function (Solid Curve).

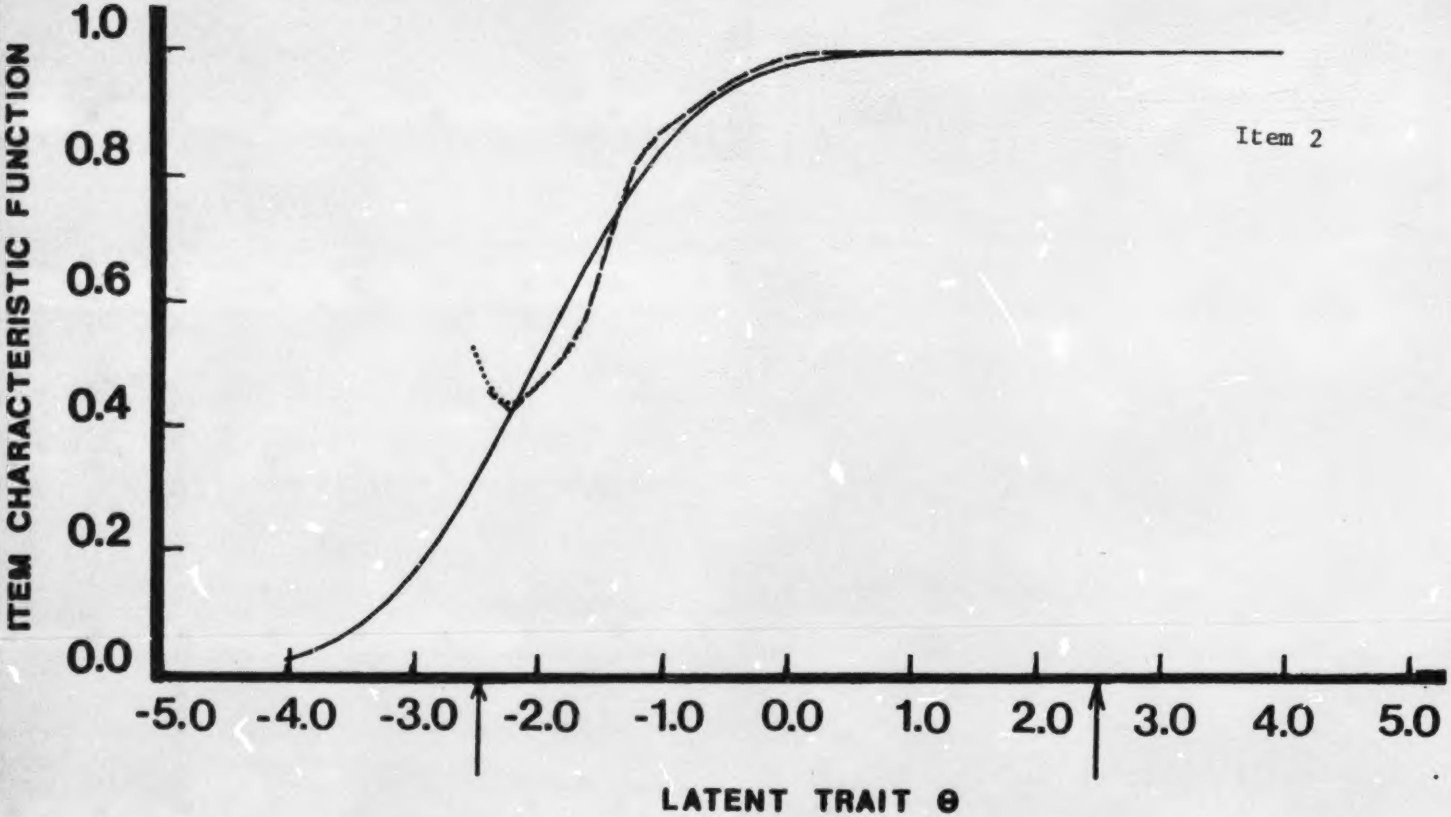


FIGURE 4-1 (Continued): Subtest 1, Criterion Item Characteristic Function.

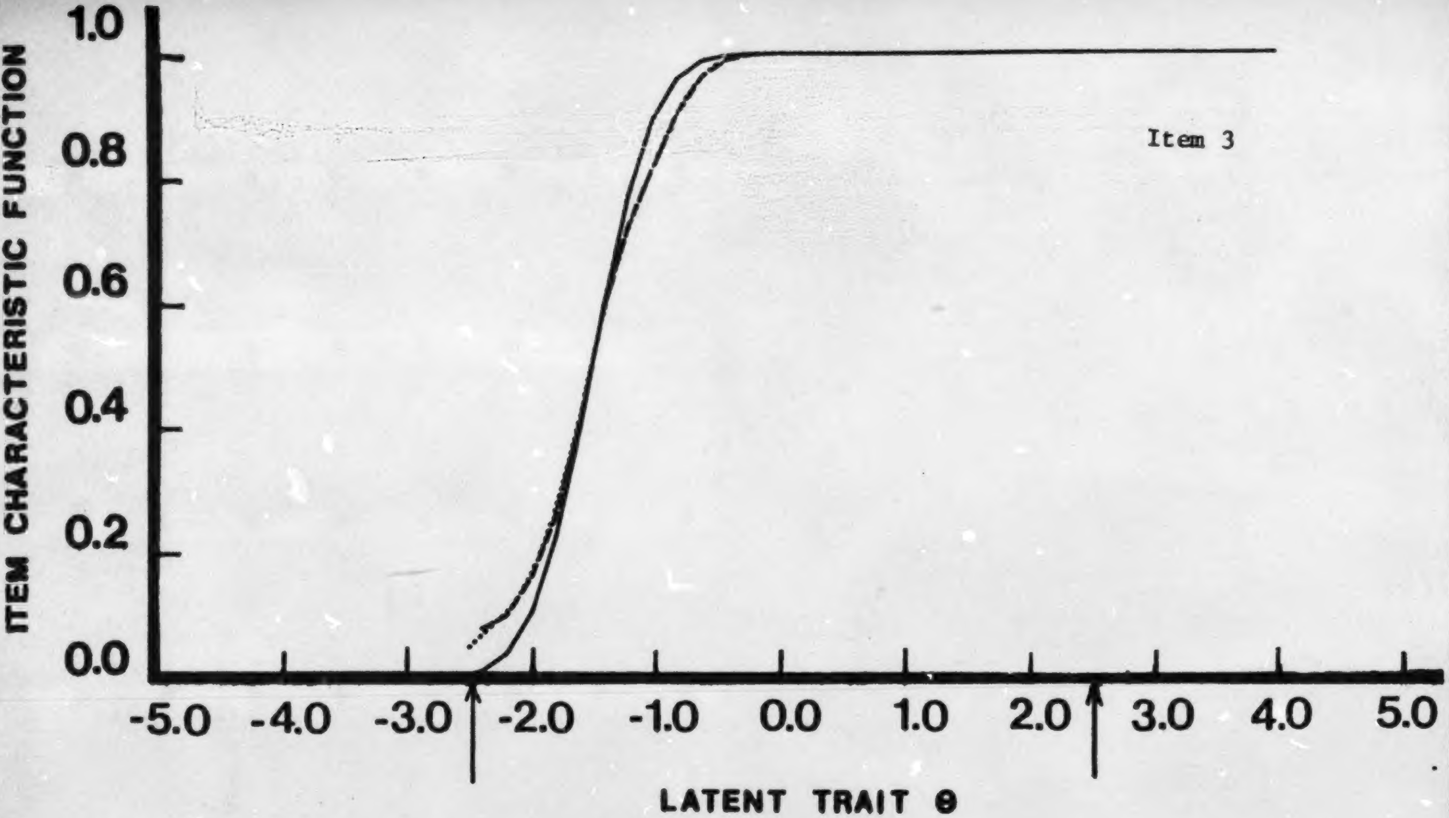


FIGURE 4-1 (Continued): Subtest 1, Criterion Item Characteristic Function.

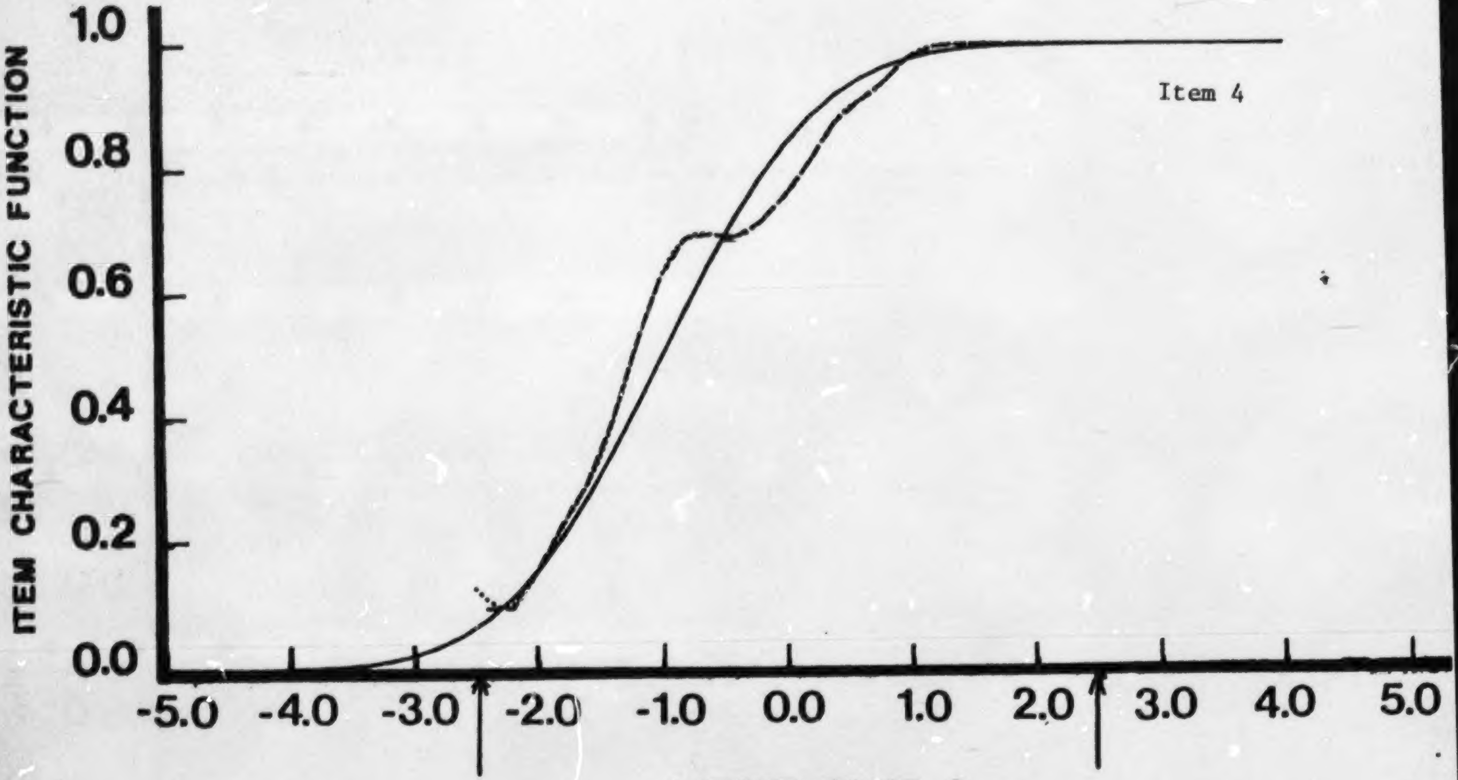


FIGURE 4-1 (Continued): Subtest 1, Criterion Item Characteristic Function.

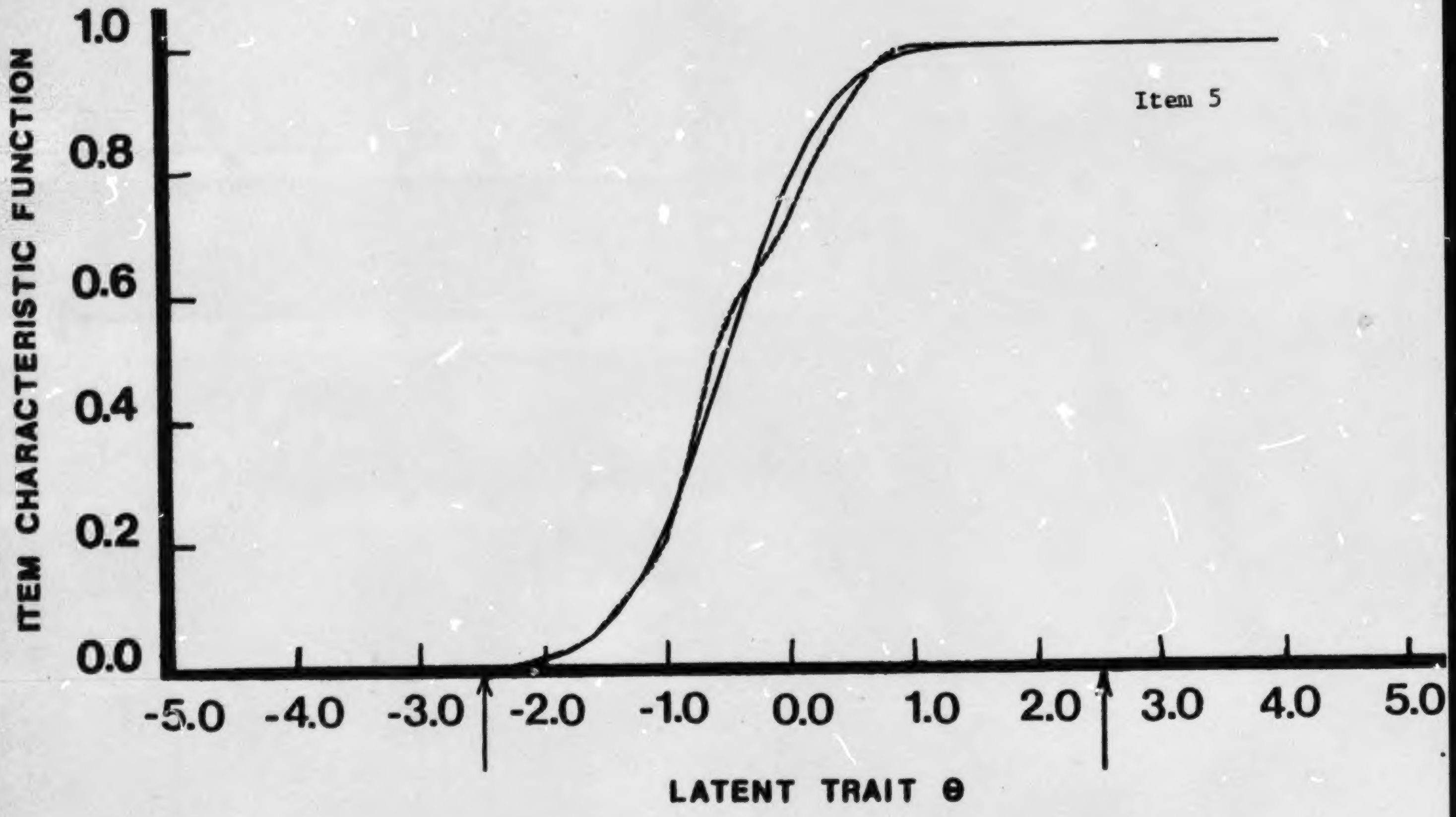


FIGURE 4-1 (Continued): Subtest 1, Criterion Item Characteristic Function.

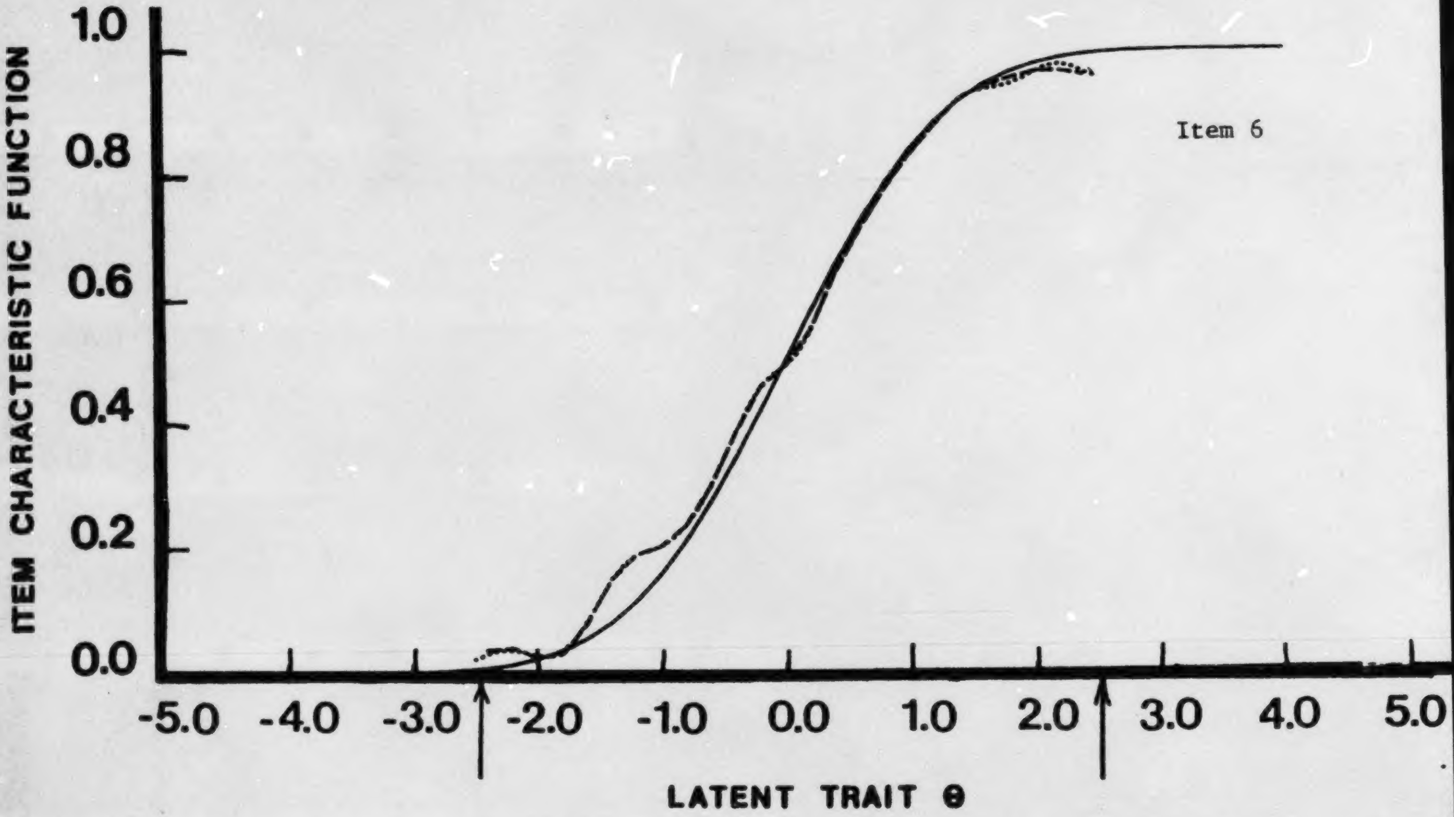


FIGURE 4-1 (Continued): Subtest 1, Criterion Item Characteristic Function.

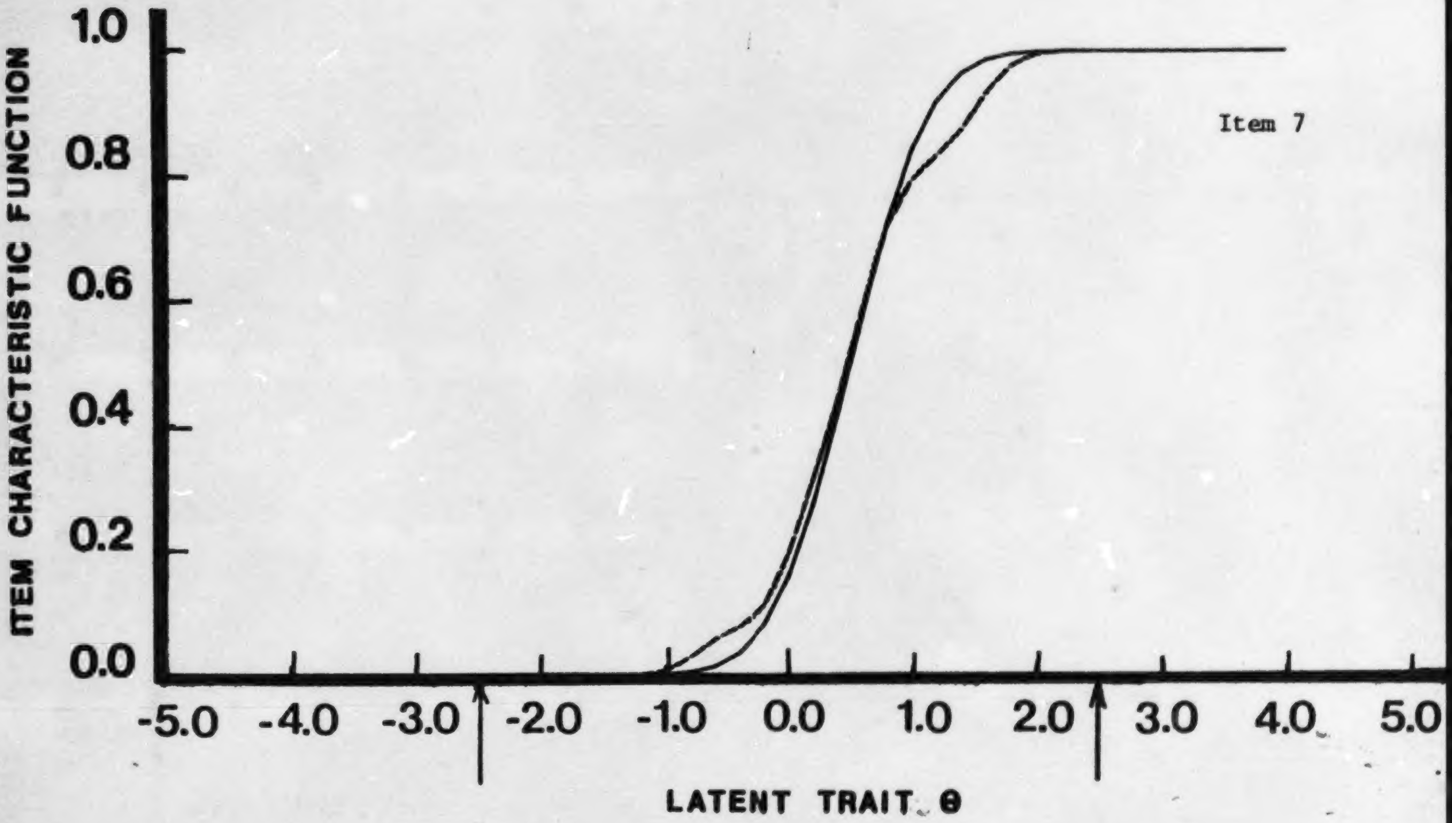
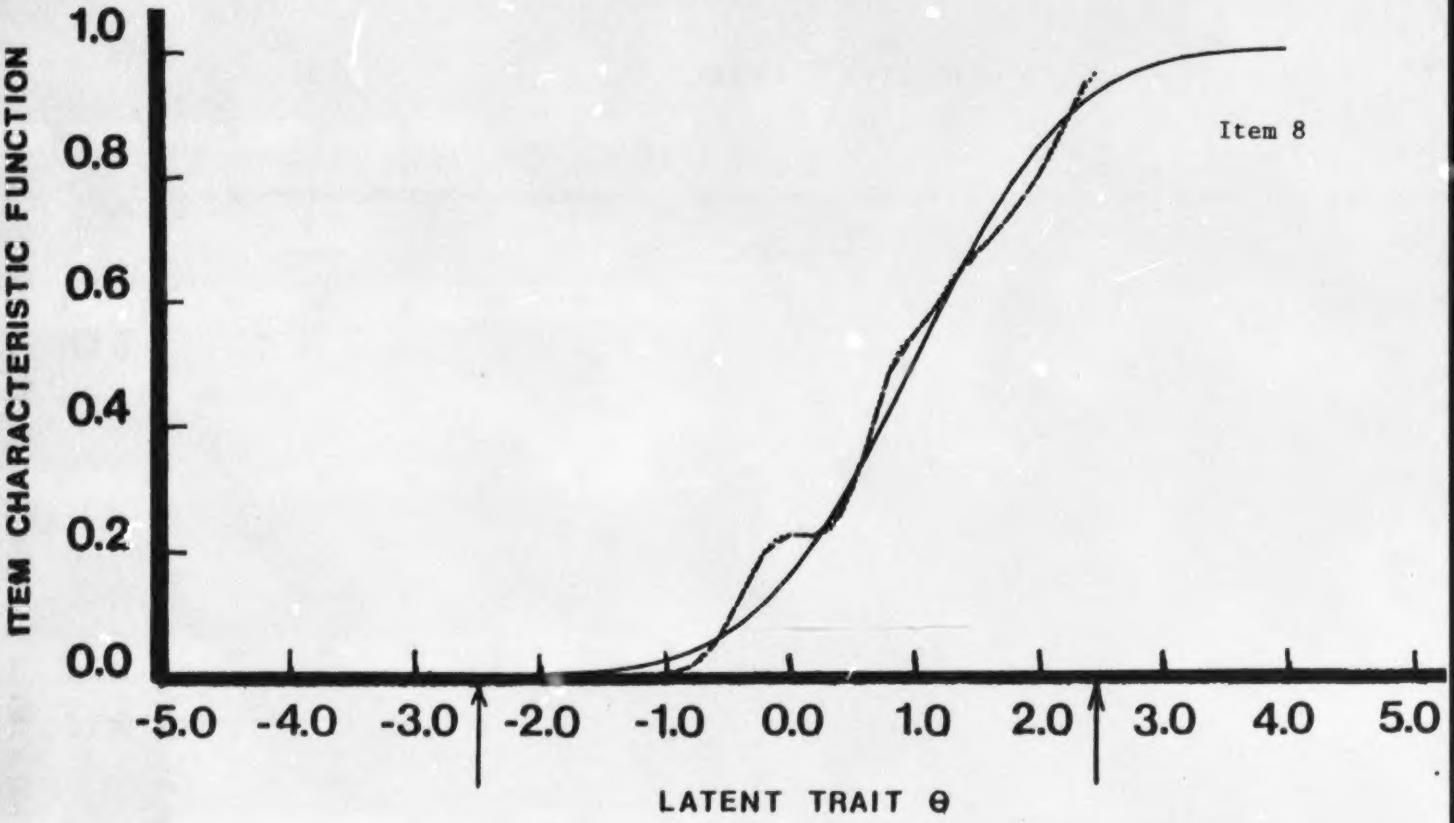


FIGURE 4-1 (Continued): Subtest 1, Criterion Item Characteristic Function.


1.0
0.8
0.6
0.4
0.2
0.0
ITEM CHARACTERISTIC FUNCTION
Item 8
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
LATENT TRAIT θ

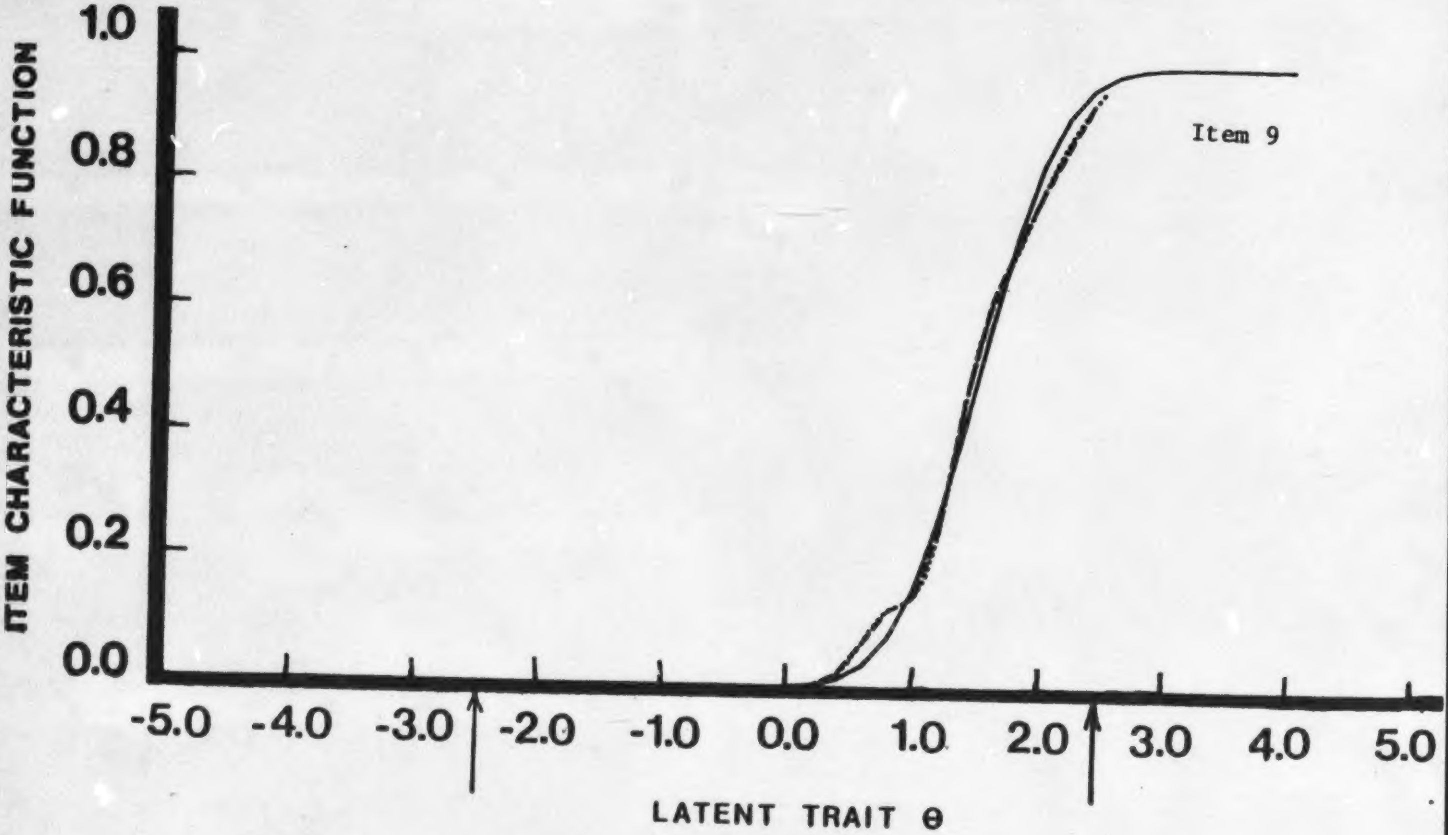


FIGURE 4-1 (Continued): Subtest 1, Criterion Item Characteristic Function.

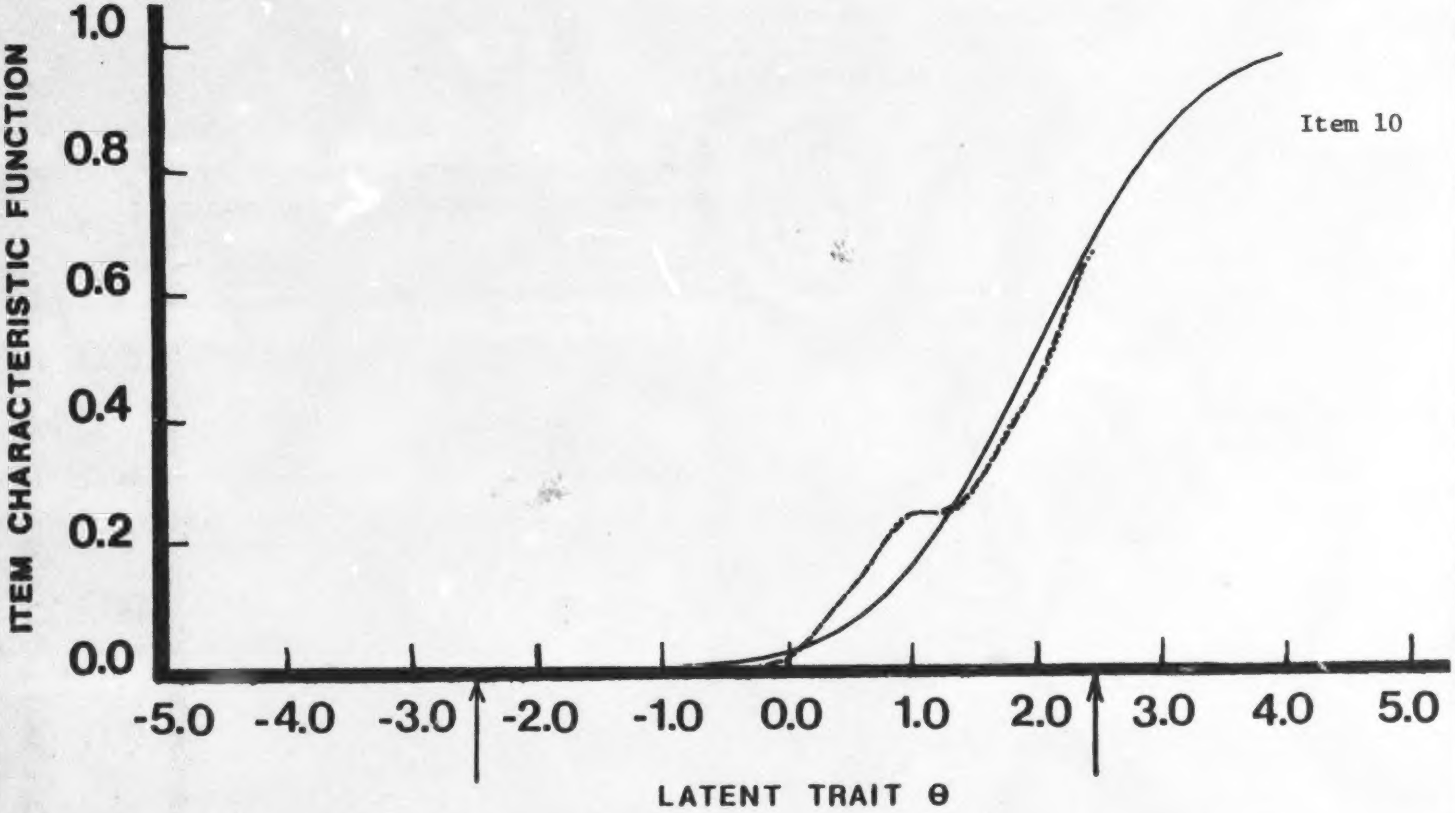


FIGURE 4-1 (Continued): Subtest 1, Criterion Item Characteristic Function.

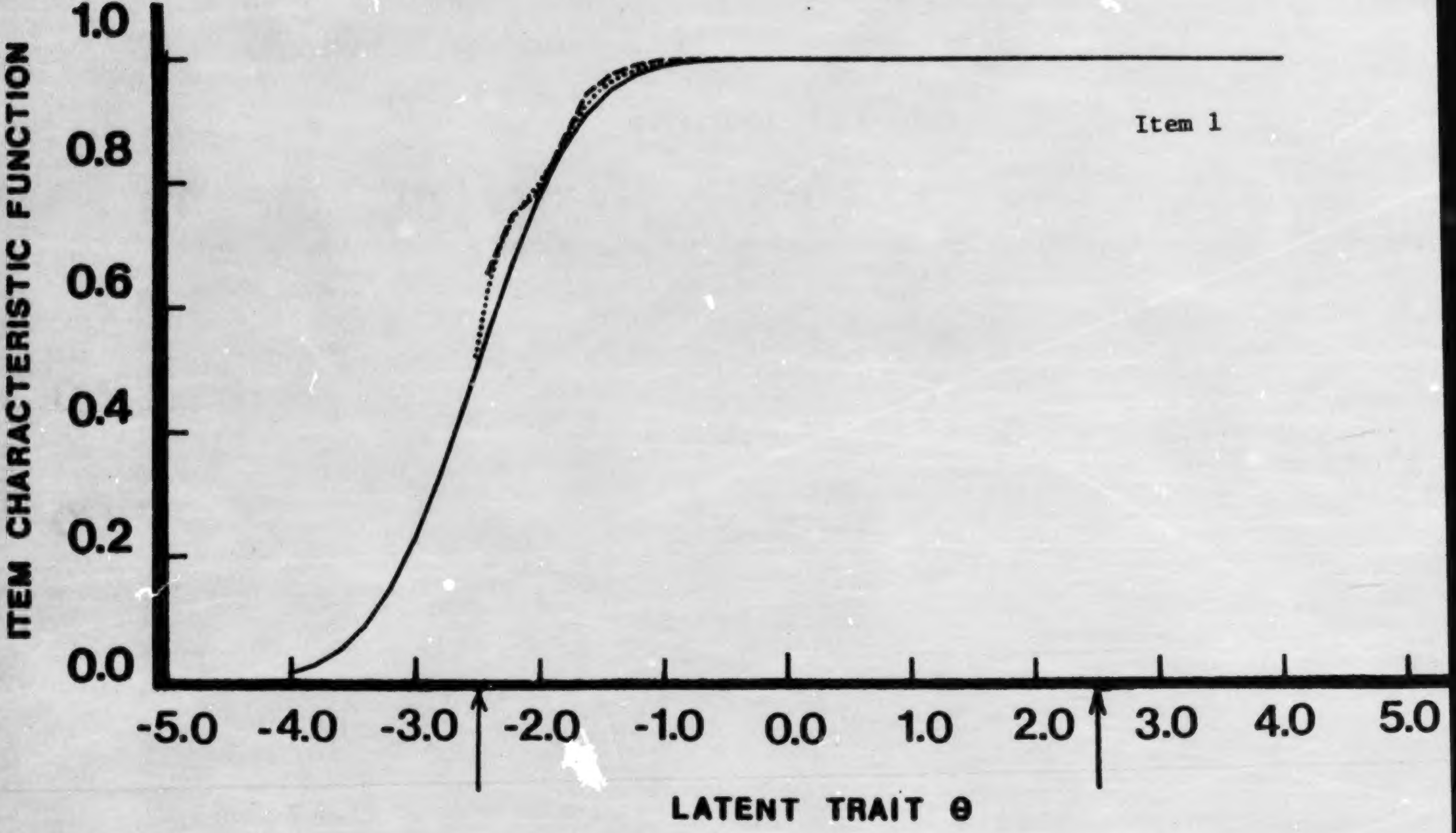


FIGURE 4-2

Criterion Item Characteristic Function Based Upon Subtest 2 (Dotted Curve), In Comparison With the One Obtained Upon the Original Old Test (Dashed Curve) And The Theoretical Item Characteristic Function (Solid Curve).

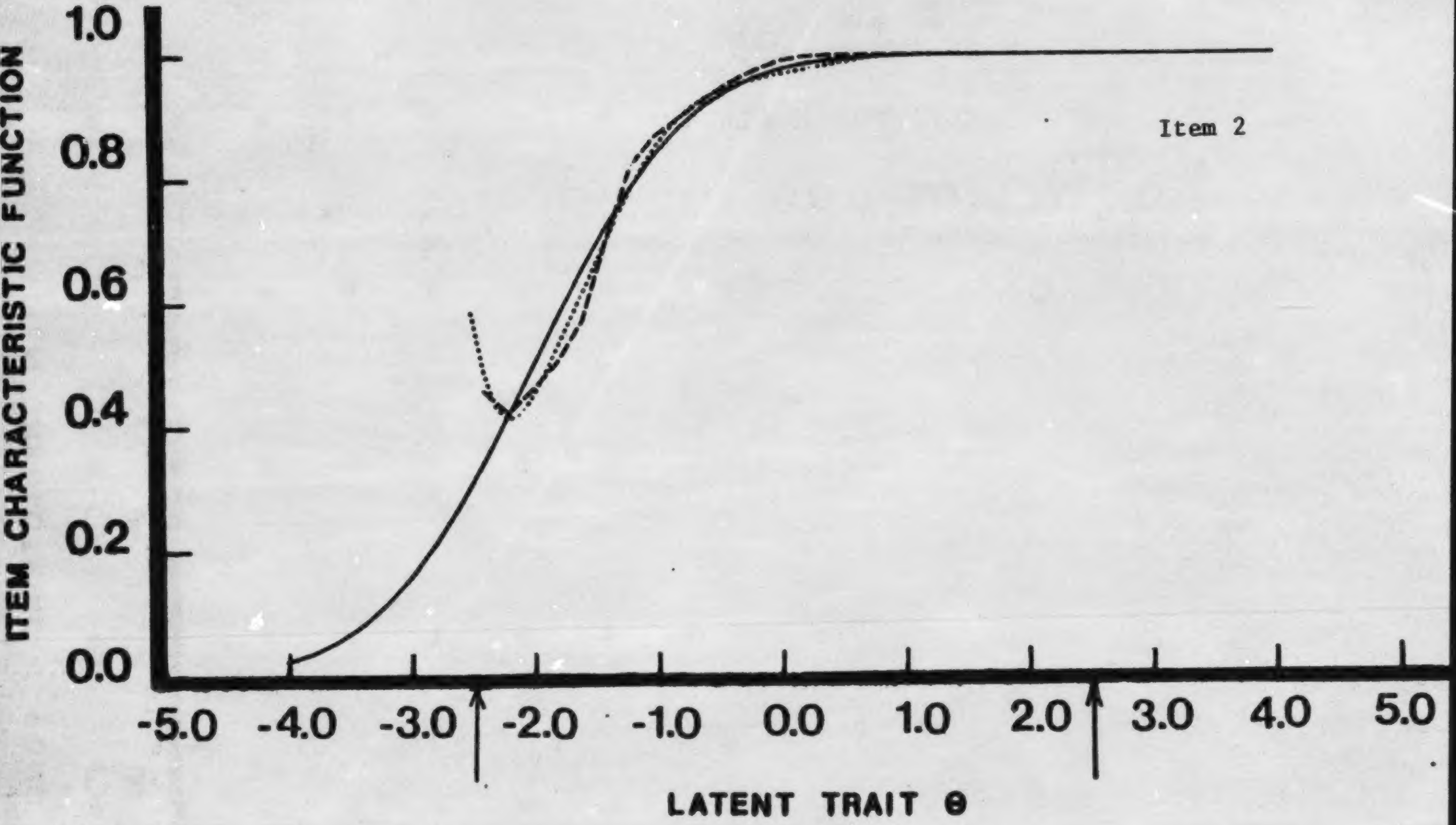


FIGURE 4-2 (Continued): Subtest 2, Criterion Item Characteristic Function.

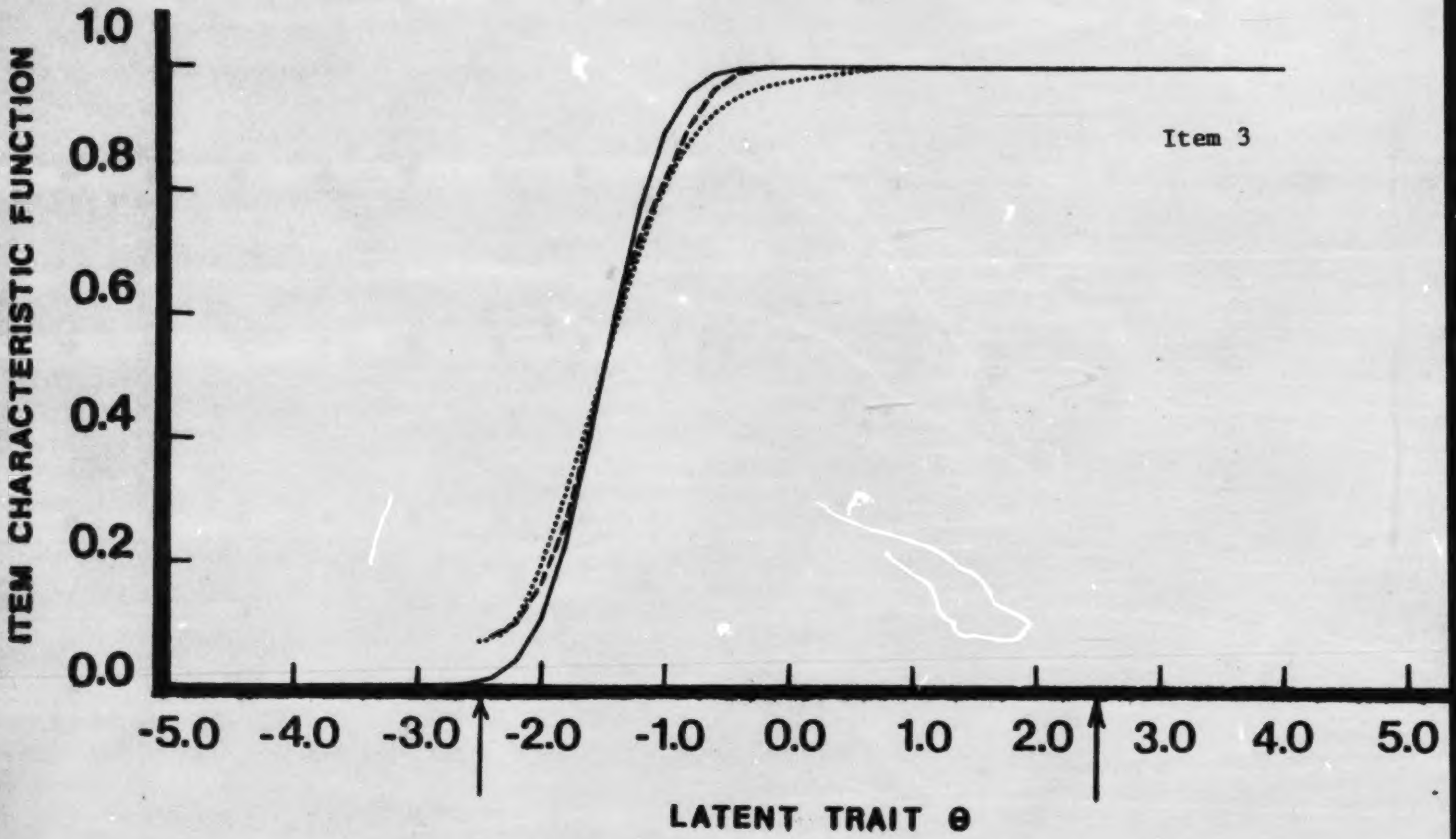


FIGURE 4-2 (Continued): Subtest 2, Criterion Item Charactersitic Function.

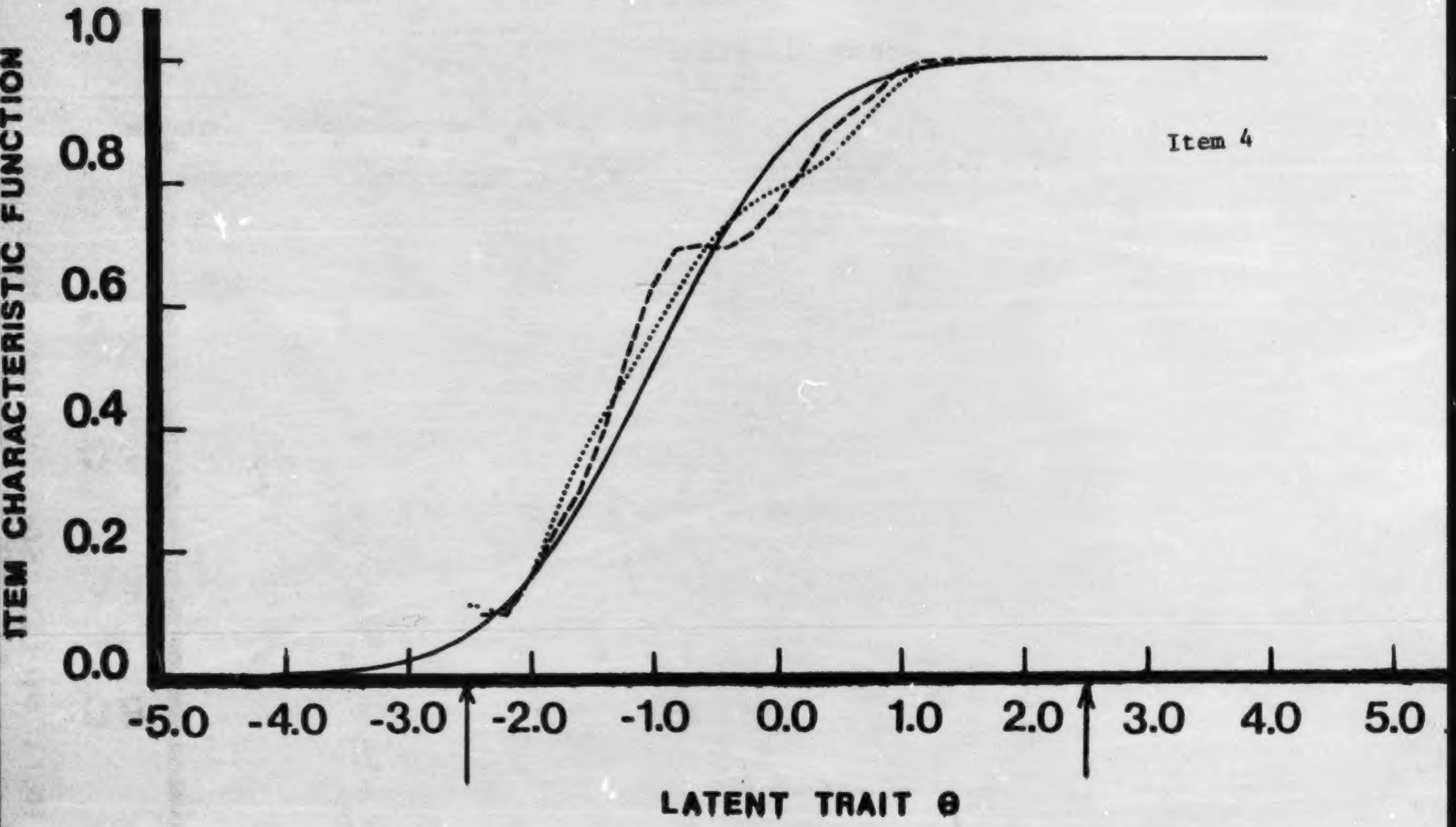


FIGURE 4-2 (Continued): Subtest 2, Criterion Item Charactersitic Function.

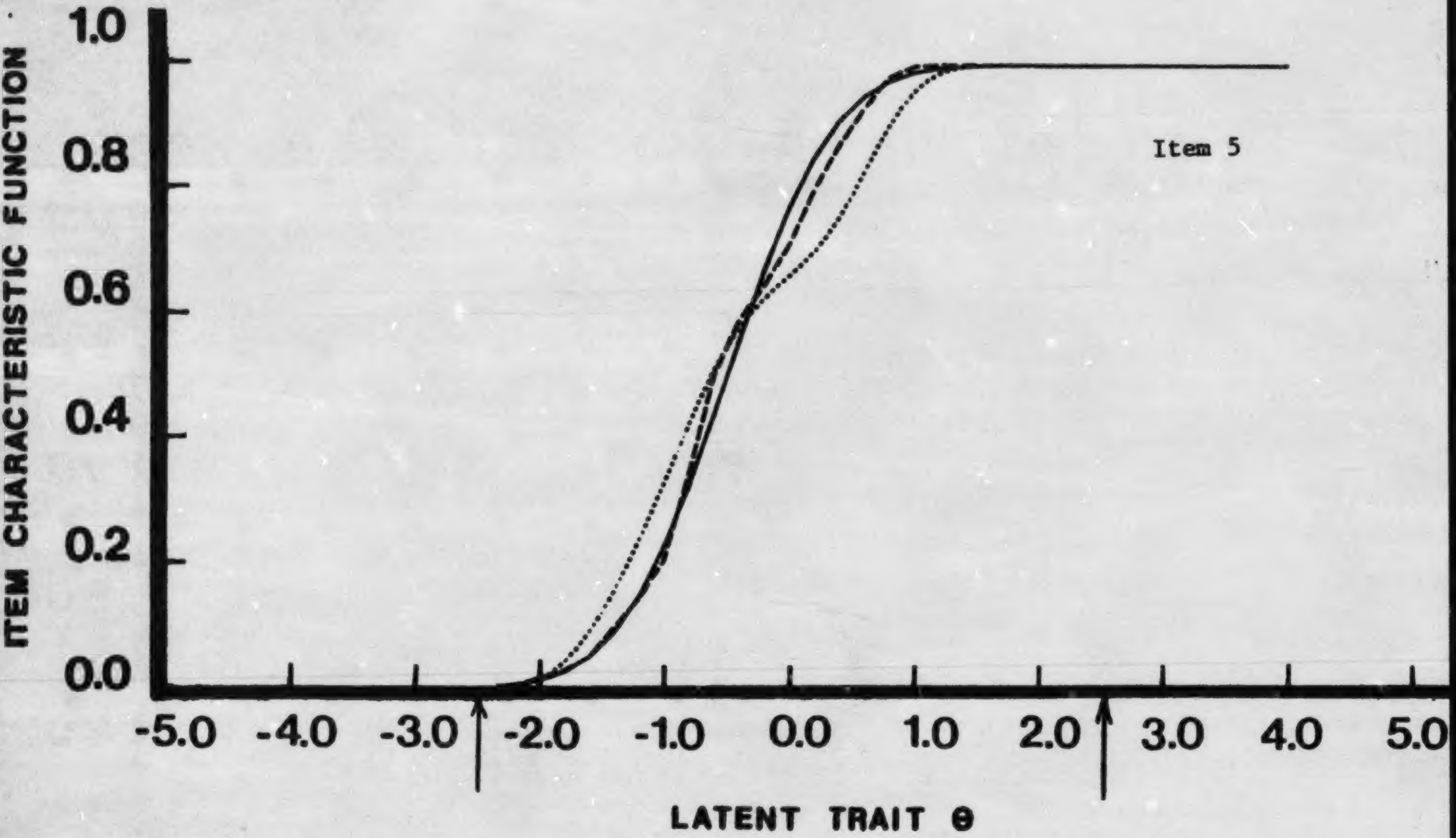


FIGURE 4-2 (Continued): Subtest 2, Criterion Item Characteristic Function.

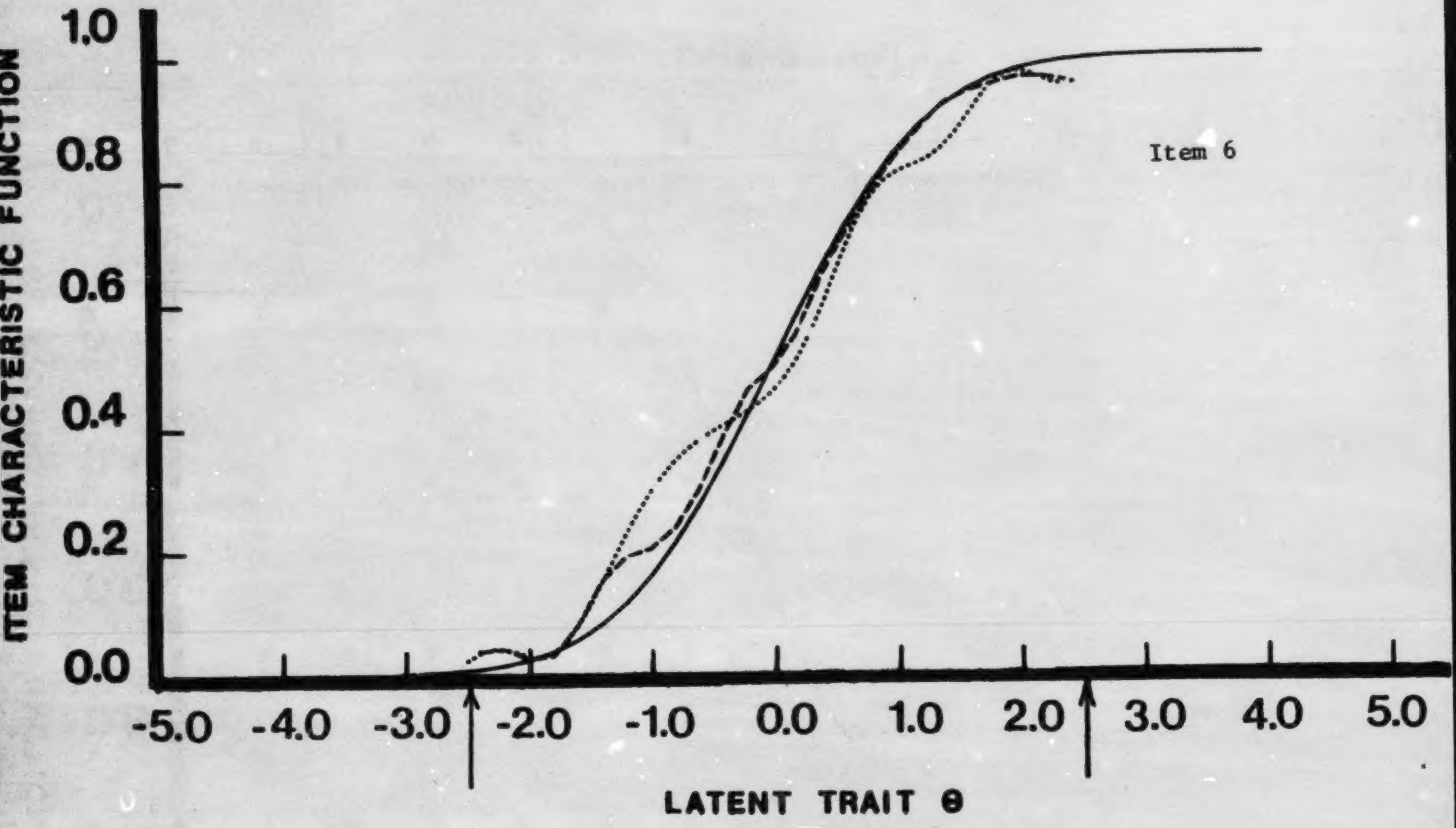


FIGURE 4-2 (Continued): Subtest 2, Criterion Item Characteristic Function.

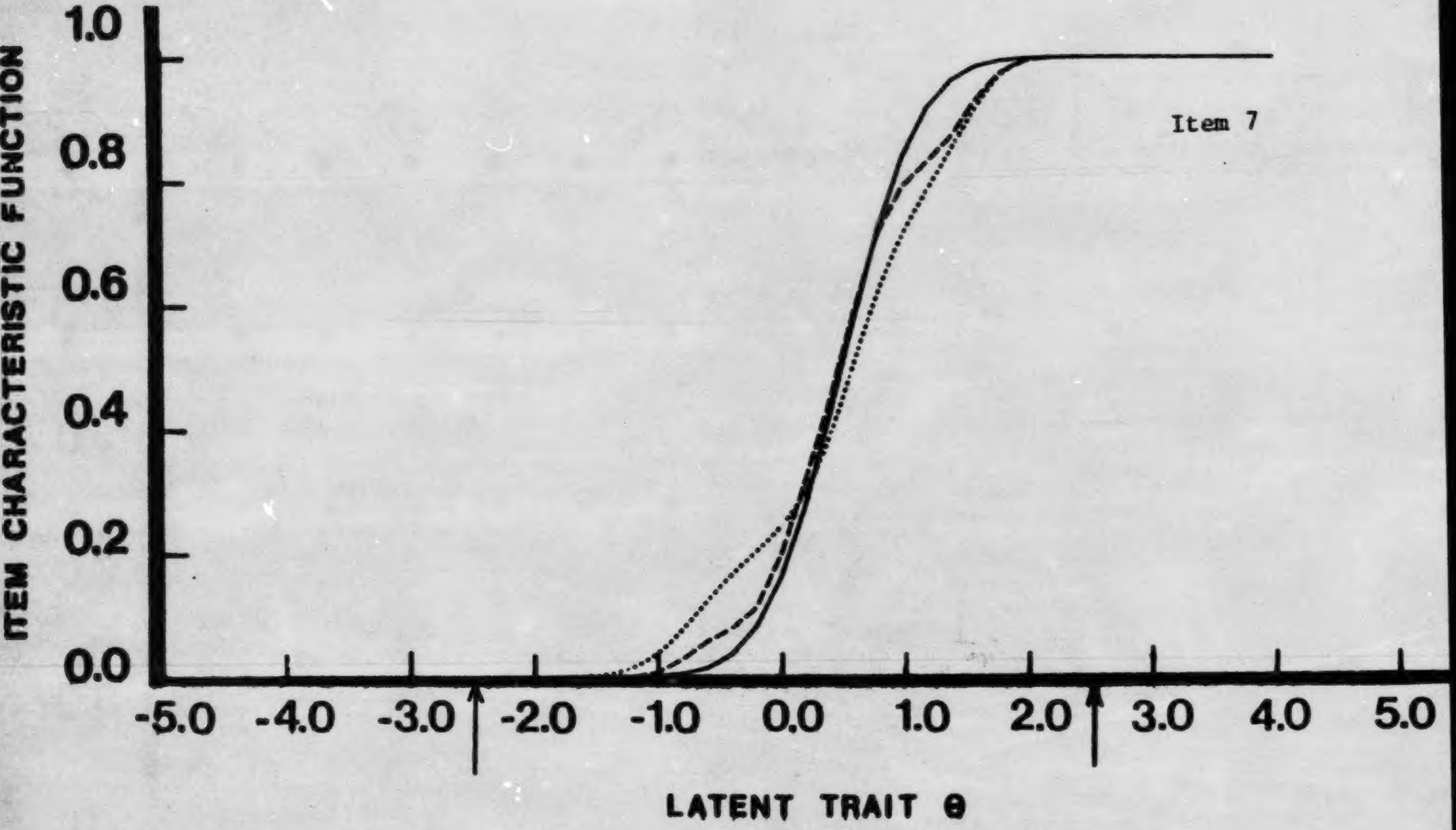


FIGURE 4-2 (Continued): Subtest 2, Criterion Item Characteristic Function.

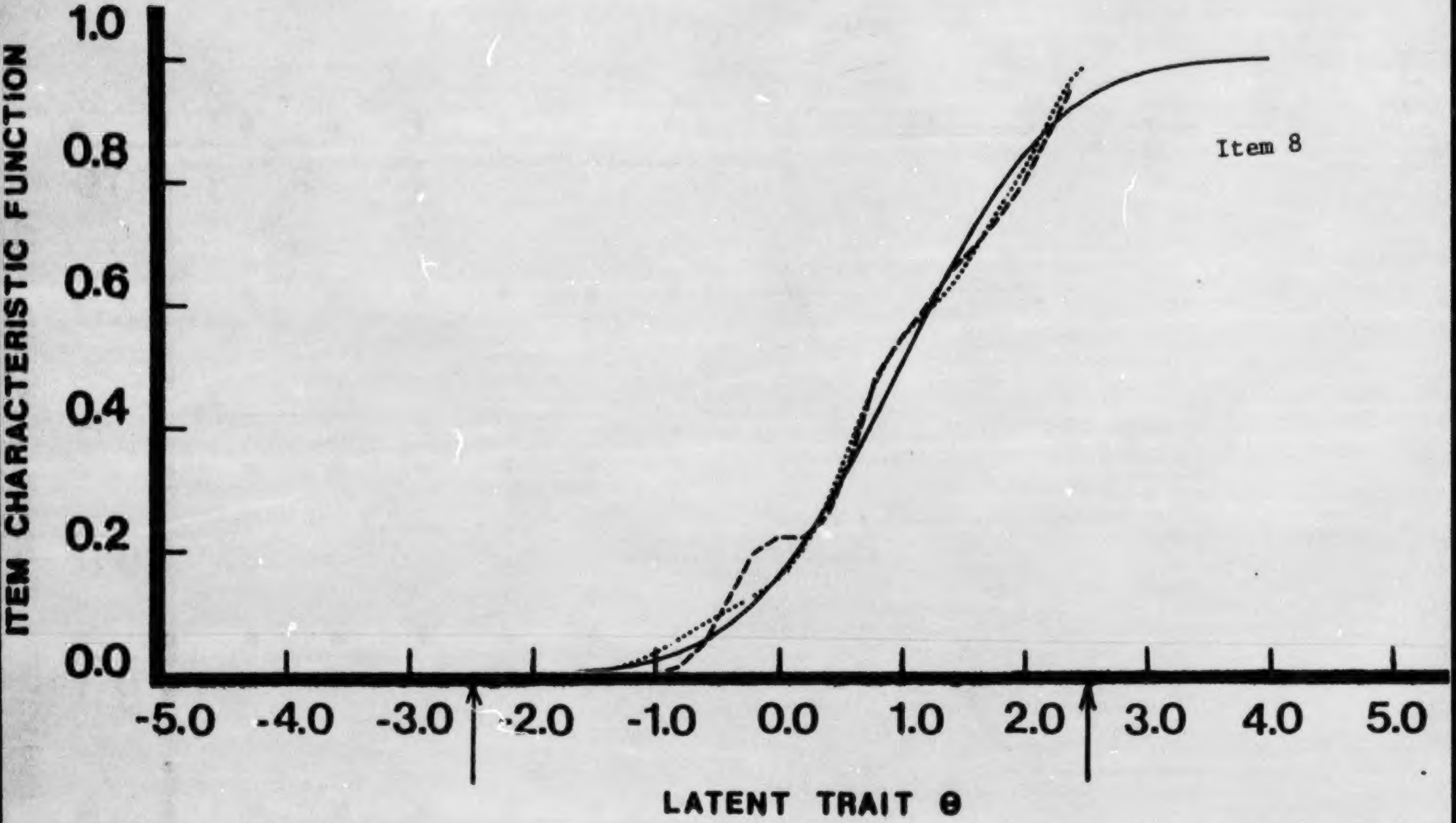


FIGURE 4-2 (Continued): Subtest 2, Criterion Item Characteristic Function.

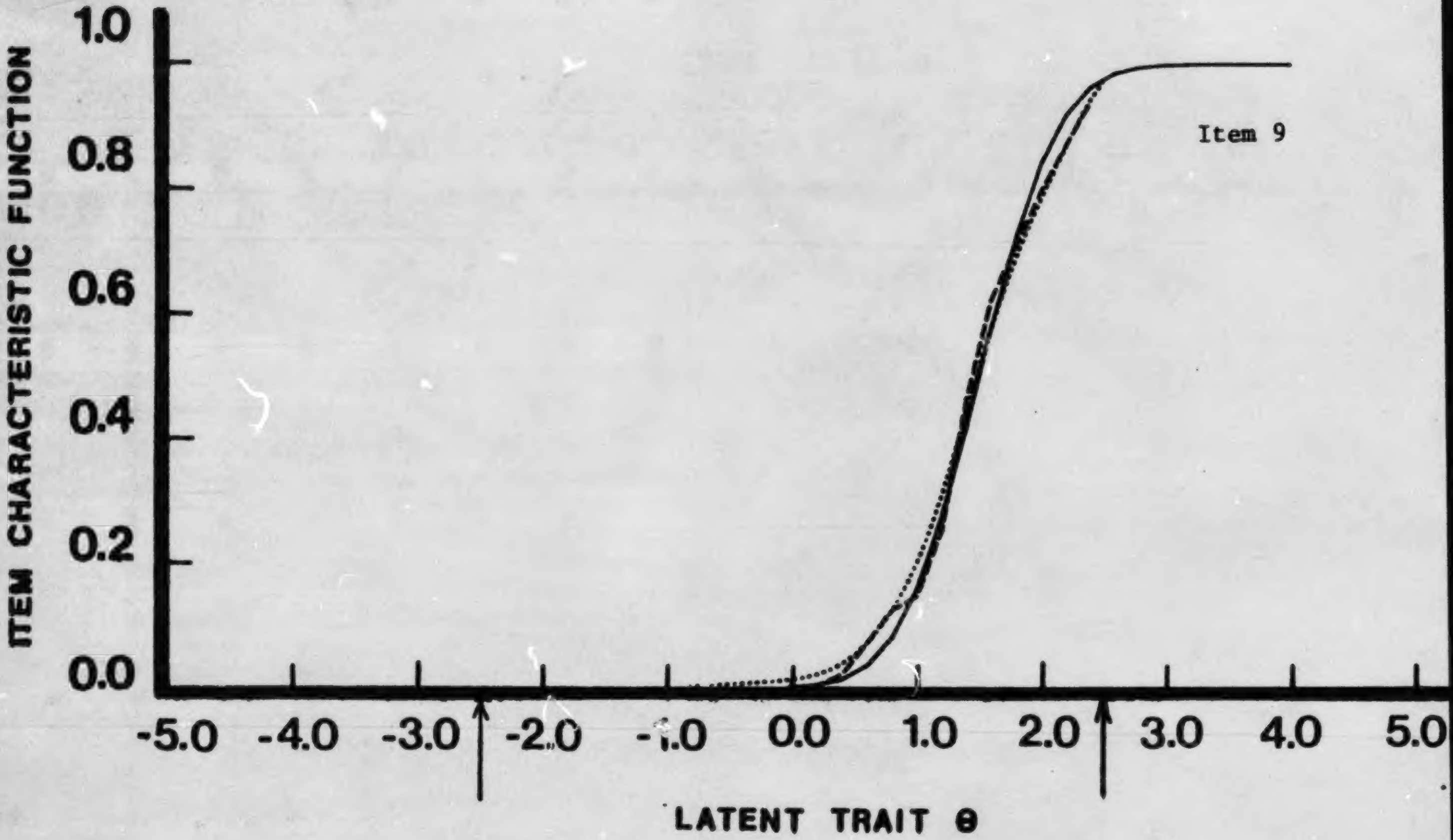


FIGURE 4-2 (Continued): Subtest 2, Criterion Item Characteristic Function.

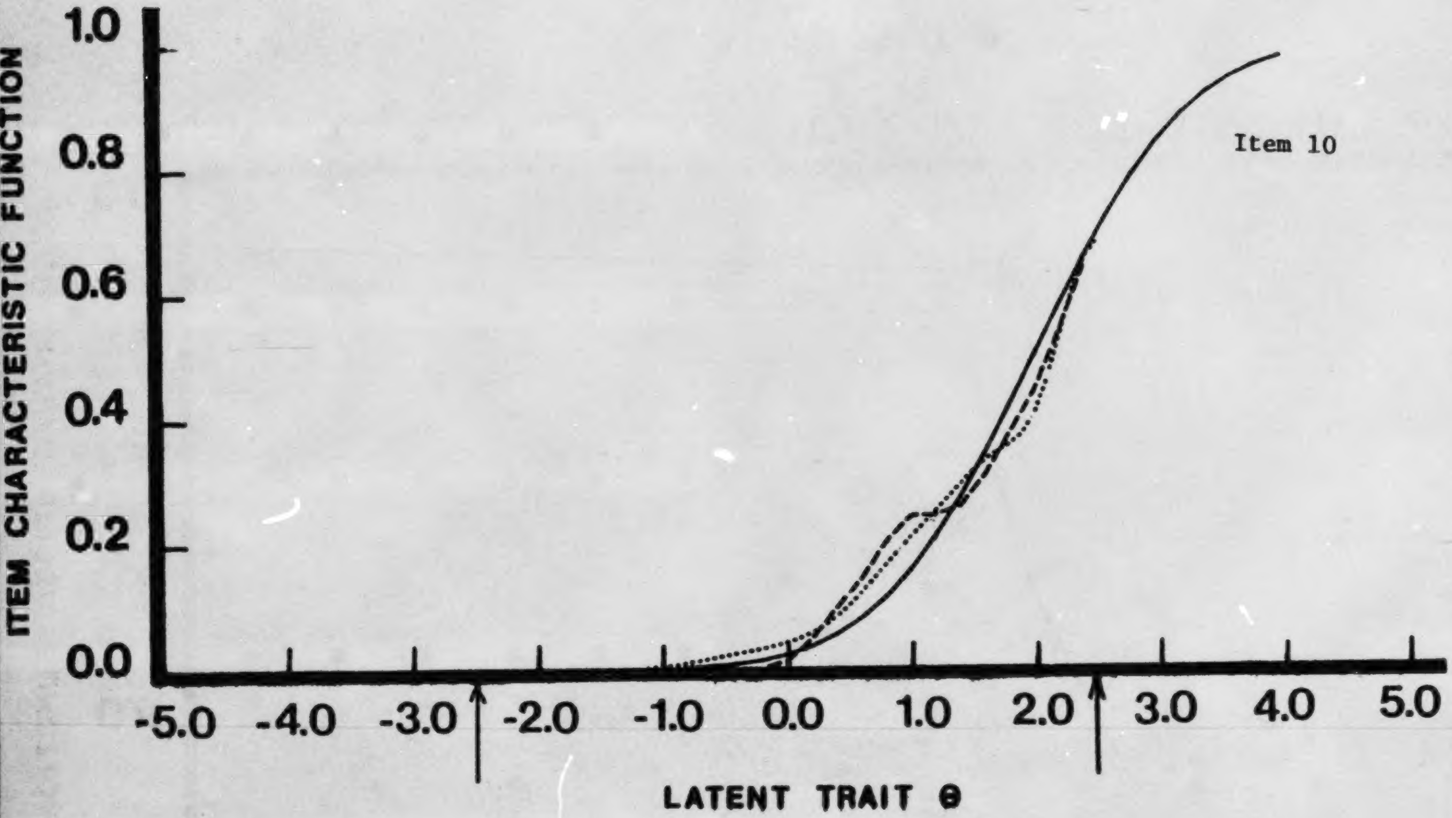


FIGURE 4-2 (Continued): Subtest 2, Criterion Item Characteristic Function.

yet these resultant criterion item characteristic functions are still close enough to the corresponding theoretical item characteristic functions. It should be noted that there exists a striking resemblance between the set of criterion item characteristic functions and the two sets of the estimated item characteristic functions obtained by the Normal Approach Method, which are shown by dotted curves in Figures 2-7 and 2-8 as Degree 3 and 4 Cases, respectively.

These findings have answered our question concerning the reason for the differences in the two sets of the estimated item characteristic functions obtained upon Subtest 1 and 2, respectively, by the Normal Approach Method, which was raised in Chapter 3. It is obvious that these differences are intrinsic in the natures of the two subtests. Normal Approach Method has provided us with practically the best possible results we can expect with the present set of data.

It is also interesting to note that for such easy items as items 1 and 2, and for such difficult items as items 8 and 10, the criterion item characteristic functions obtained upon Subtest 2 are closer to the corresponding theoretical item characteristic functions, in comparison with those obtained upon Subtest 1, whereas for such items of intermediate difficulties as items 5, 6 and 7 those obtained upon Subtest 1 are closer to the corresponding theoretical item characteristic functions than those obtained upon Subtest 2. This fact is obviously related with the differences in the square roots of the test information functions for the two subtests which are presented in Figures 2-1 and 2-2, respectively.

The above observations are in congruence with the fact that the improvement in the approximation to the square root of the test information function for Subtest 2 by the use of the three polynomials did not noticeably ameliorate the estimated item characteristic functions, which we observed in Chapter 3. We must further infer that neither will the improvement in the approximation to the marginal density function, $g(\tau)$, ameliorate the estimation. To examine this, the true density function, $g(\tau)$, was obtained through (4.10), for each of Subtests 1 and 2. Figure 4-3 presents the resultant density function, $g(\tau)$, obtained upon Subtest 1. In this figure, the density function is plotted against the five hundred values of τ , and the histogram for the actual frequency distribution of τ is also shown. If we compare this true density function, $g(\tau)$, with the polynomial of degree 4, which approximates $g(\tau)$ and is presented in the second graph of Figure 2-3, we can see that this polynomial obtained by the method of moments is a good approximation to the true density function. In fact, this polynomial of degree 4 is closer to the true density function than it is to the other polynomial of degree 3, which was also obtained by the method of moments and is shown in the first graph of Figure 2-3 as another approximation to the true density function, $g(\tau)$. Considering the fact that the estimated item characteristic functions in Degree 3 and 4 Cases are practically identical with each other for each of the ten binary items, it is

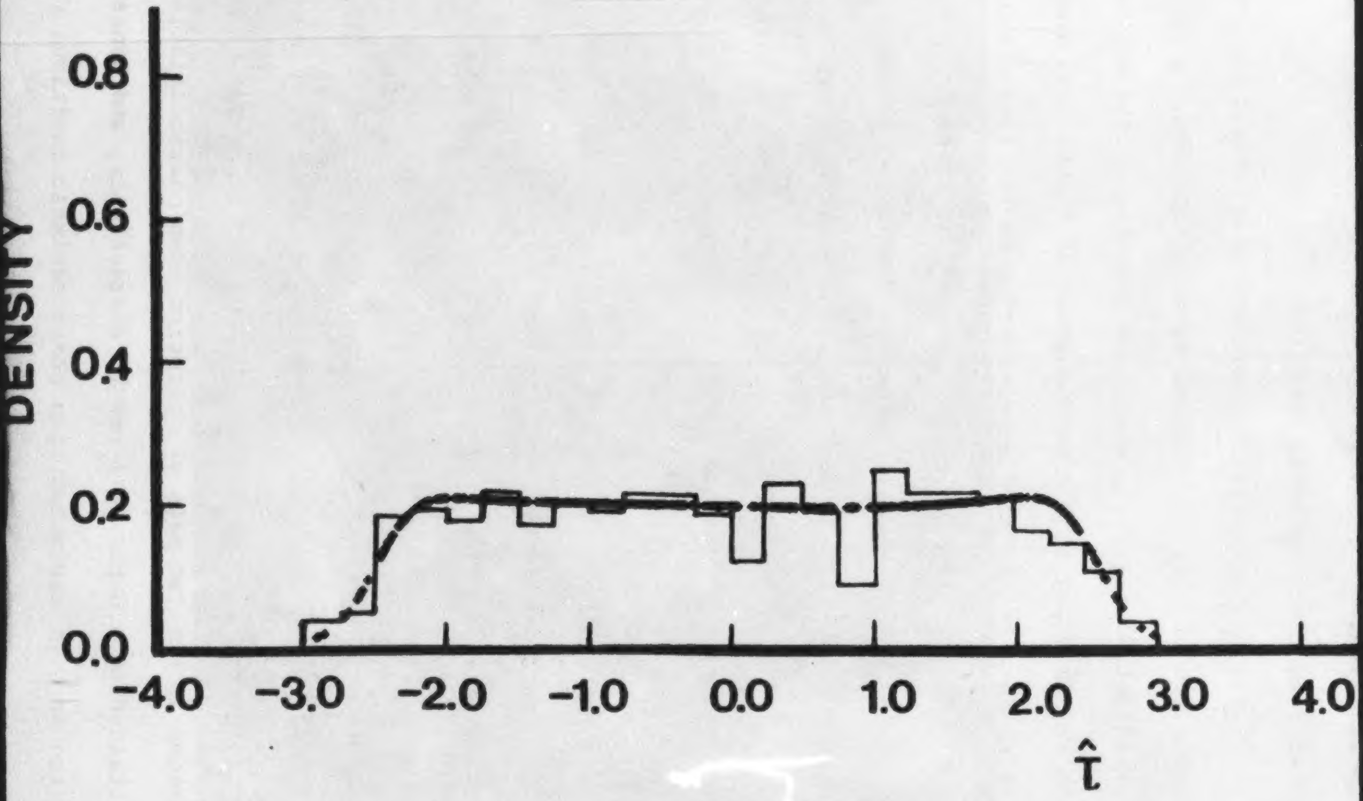


FIGURE 4-3

True Density Function, $g(\hat{\tau})$, of $\hat{\tau}$, Plotted for the Five Hundred Examinees in Comparison with the Actual Frequency Distribution of $\hat{\tau}$: A Result Based on Subtest 1.

obvious that the further improvement in the approximation to the density function, $g(\hat{\tau})$, will not substantially affect the resultant estimated item characteristic functions.

Figure 4-4 presents the corresponding result for Subtest 2. The comparison of this theoretical density function, $g(\hat{\tau})$, with the polynomial of degree 4, which was obtained by the method of moments and is shown in the second graph of Figure 2-4, indicates that the latter is a fairly good approximation to the true density function, $g(\hat{\tau})$, even though they are not as close to each other as they are for Subtest 1. The same negative conclusion as the one drawn for Subtest 1 also holds for Subtest 2, therefore, concerning the possibility of improvement in the accuracy of estimation to the item characteristic function by a closer approximation to the density function, $g(\hat{\tau})$.

The density function, $f^*(\tau)$, of the transformed latent trait τ was obtained by (4.8) for both Subtests 1 and 2. These two results are shown in Figures 4-5 and 4-6, respectively, by solid curves, in comparison with the density function, $f(\theta)$, of the original latent trait θ , which is drawn by dotted lines.

Comparison of these two figures, together with the comparison of the two graphs in Figure A-2-1 in Appendix II, explains the differences in the two sets of criterion item characteristics, which were obtained upon Subtests 1 and 2, respectively, and hence those in the two sets of estimated item characteristic functions obtained

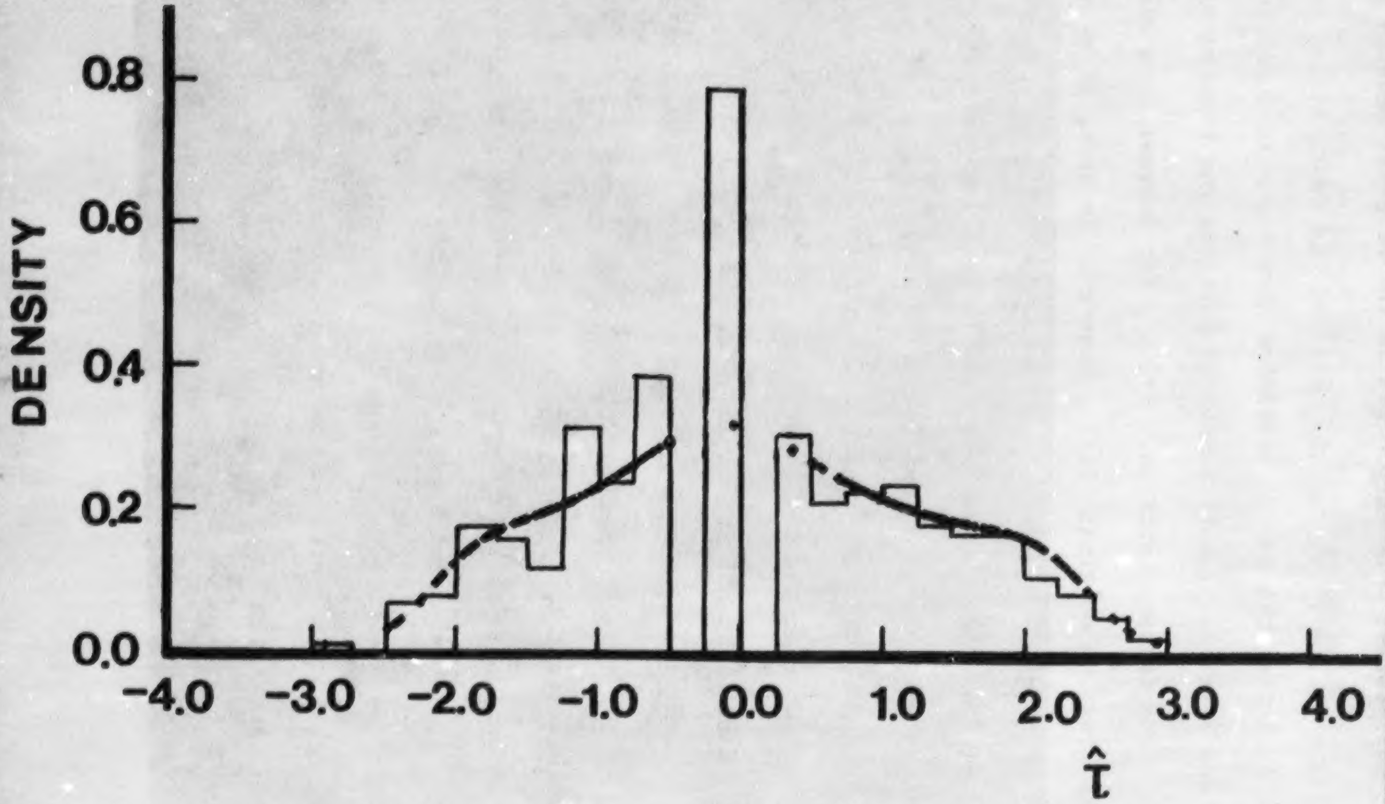


FIGURE 4-4

True Density Function, $g(\hat{\tau})$, of $\hat{\tau}$, Plotted for the Five Hundred Examinees in Comparison with the Actual Frequency Distribution of $\hat{\tau}$: A Result Based on Subtest 2.

by the Normal Approach Method. The fact that the density function, $f^*(\tau)$, for Subtest 1 is very close to the original density function, $f(\theta)$, which is the $f^*(\tau)$ for the original Old Test, is in congruence with the fact that the set of criterion item characteristic functions for Subtest 1 is practically identical with the one for the original Old Test. The density function, $f^*(\tau)$, for Subtest 2 is substantially different from the original $f(\theta)$, however. In fact, if we use $C = 4.5$ instead of $C = 3.5$, this curve for $f^*(\tau)$, will become even steeper and the interval of τ will grow even narrower. If we visualize the criterion item characteristic function of each of the ten binary items which is plotted against τ , instead of θ , then we can see that the resultant function will look as close to the theoretical item characteristic function, which is also plotted against τ , as the corresponding criterion item characteristic function based upon the original Old Test is to the theoretical item characteristic function, which is plotted against θ . We will no longer feel, therefore, that the criterion item characteristic functions for items of intermediate difficulties, which are based upon Subtest 2, are worse than those obtained upon the original Old Test, nor will we perceive the opposite relationship for either easy or difficult items.

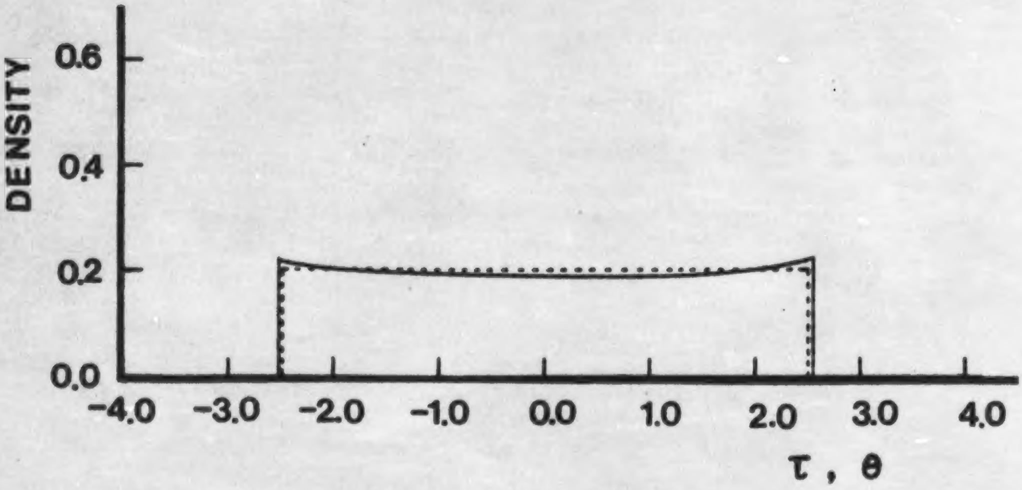


FIGURE 4-5

Density Function, $f^*(\tau)$, of τ Transformed from θ by the Polynomial of Degree 8 Which Was Obtained for Subtest 1 (Solid Curve), in Contrast to the Original Density Function $f(\theta)$ (Dotted Curve).

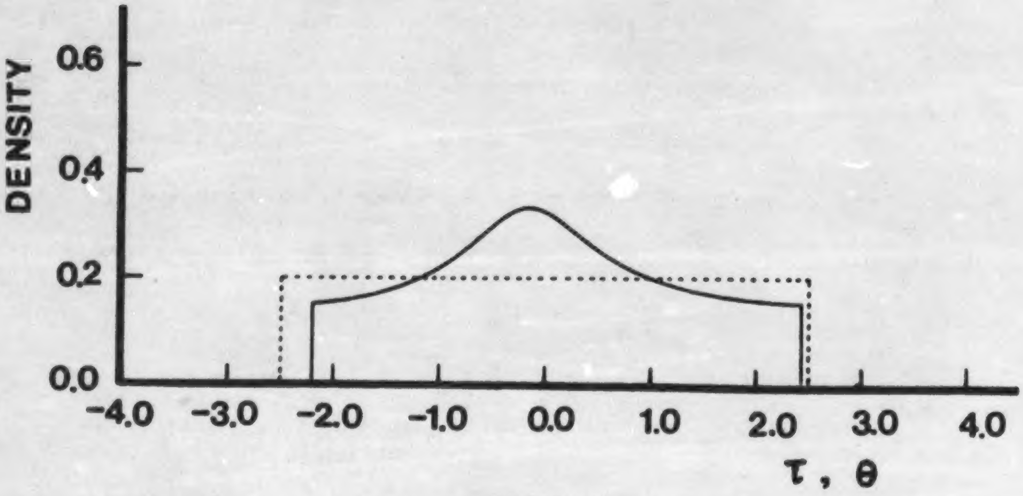


FIGURE 4-6

Density Function, $f^*(\tau)$, of τ Transformed from θ by the Polynomial of Degree 8 Which Was Obtained for Subtest 2 (Solid Curve), in Contrast to the Original Density Function $f(\theta)$ (Dotted Curve).

V Discussion and Conclusions

It has been shown that the method of estimating the operating characteristics of discrete item responses when the amount of test information of the Old Test is not constant works well with our data of five hundred hypothetical examinees in estimating the item characteristic functions of the ten binary items. In the present study, we only used the Simple Sum Procedure of the Conditional P.D.F. Approach combined with the Normal Approach Method. The result is encouraging, however, to the extent that we can assume it will work well using any one of the other combinations of an approach and a method, considering the fact that they are all based upon the same rationale in dealing with the non-constant test information function.

We have also found out that the nature of the set of test items used as the Old Test substantially affects the resultant estimated item characteristic functions. This is clear from the result obtained for the criterion item characteristic functions, which was discussed in Chapter 4. It is reasonable that the criterion item characteristic function for each of the ten binary items, which was obtained upon Subtest 1, turned out to be identical with the one obtained upon the original Old Test, since the square root of the test information function for Subtest 1 is similar to the one for the original Old Test for the interval of θ , (-2.5, 2.5), in which the ability levels of all the five hundred hypothetical examinees are located.

Unlike Subtest 1, however, Subtest 2 has a substantially different square root of the test information function for the same interval of θ, as we can see in Figures 2-2 and 3-3. It is reasonable, therefore, that the resultant criterion item characteristic function for each of the ten binary items obtained upon Subtest 2 is somewhat different from the one obtained upon the original Old Test.

The method of moments proved to be useful in the present method, just as it happened in many situations in our previous studies. It also turned out that both Degree 3 and 4 Cases provided us with practically identical results, just as we have observed in our previous studies.

The reduction of the number of test items from thirty-five in the original Old Test to twenty-five in both Subtests 1 and 2 proved to be acceptable, which is a desirable result for the benefit of researchers applying the method for actual data.

We can conclude, therefore, that the present method of estimating the operating characteristics of discrete item responses when the amount of test information of the Old Test is not constant is a success, at least for the kind of data like ours. It is suggested from our results that we probably need even a smaller number of test items in the Old Test than twenty-five. This should be investigated in a future research, in combination with the selection of those items.

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APPENDIX I

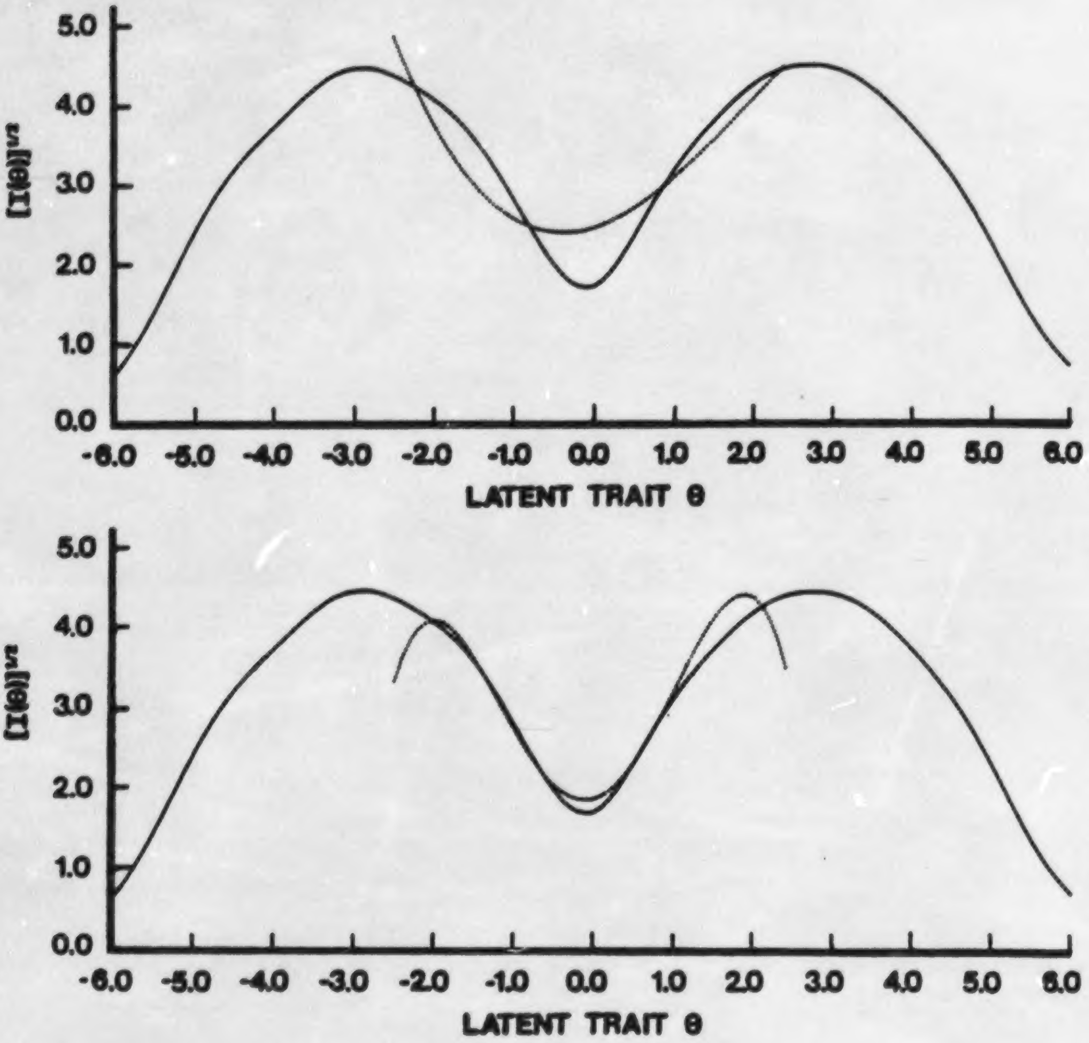
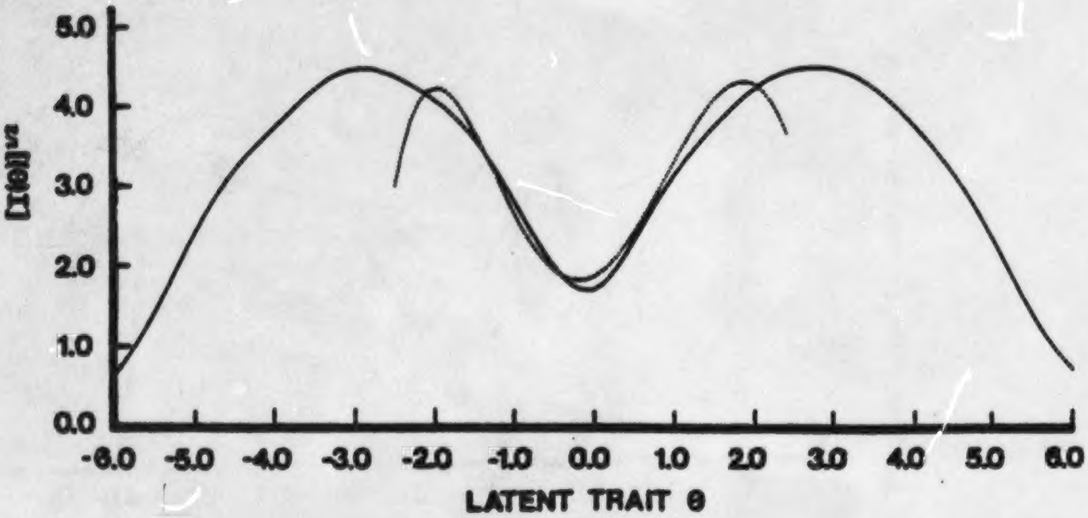
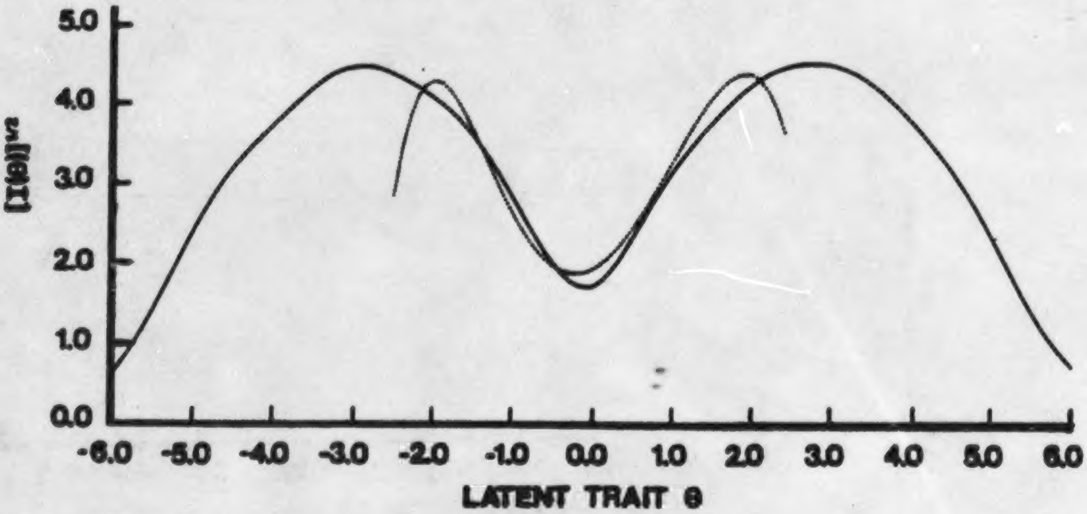
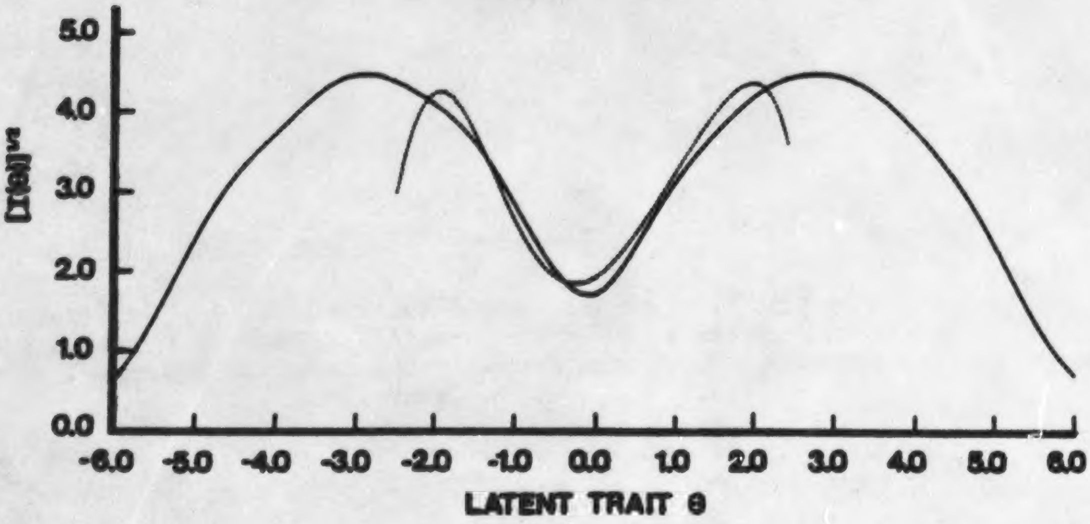


FIGURE A-1-1

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) of Subtest 2 and the Polynomials of Degrees 3 through 7 (Dotted Line), Which Were Fitted by the Method of Moments with [-2.5, 2.5] as the Interval of θ.


[I(θ)]^1/2
5.0
4.0
3.0
2.0
1.0
0.0
-6.0 -5.0 -4.0 -3.0 -2.0 -1.0 0.0 1.0 2.0 3.0 4.0 5.0 6.0
LATENT TRAIT θ


[I(θ)]^1/2
5.0
4.0
3.0
2.0
1.0
0.0
-6.0 -5.0 -4.0 -3.0 -2.0 -1.0 0.0 1.0 2.0 3.0 4.0 5.0 6.0
LATENT TRAIT θ


[I(θ)]^1/2
5.0
4.0
3.0
2.0
1.0
0.0
-6.0 -5.0 -4.0 -3.0 -2.0 -1.0 0.0 1.0 2.0 3.0 4.0 5.0 6.0
LATENT TRAIT θ

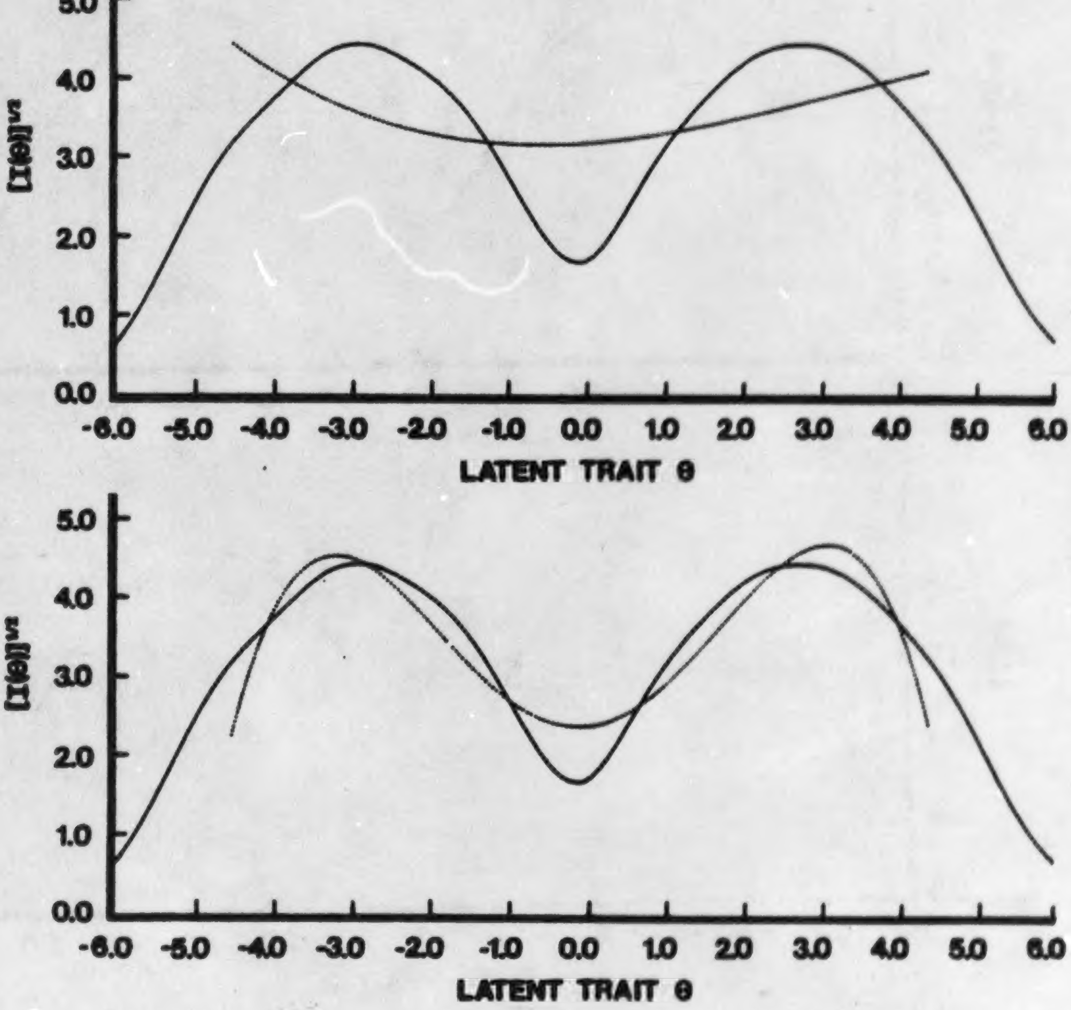
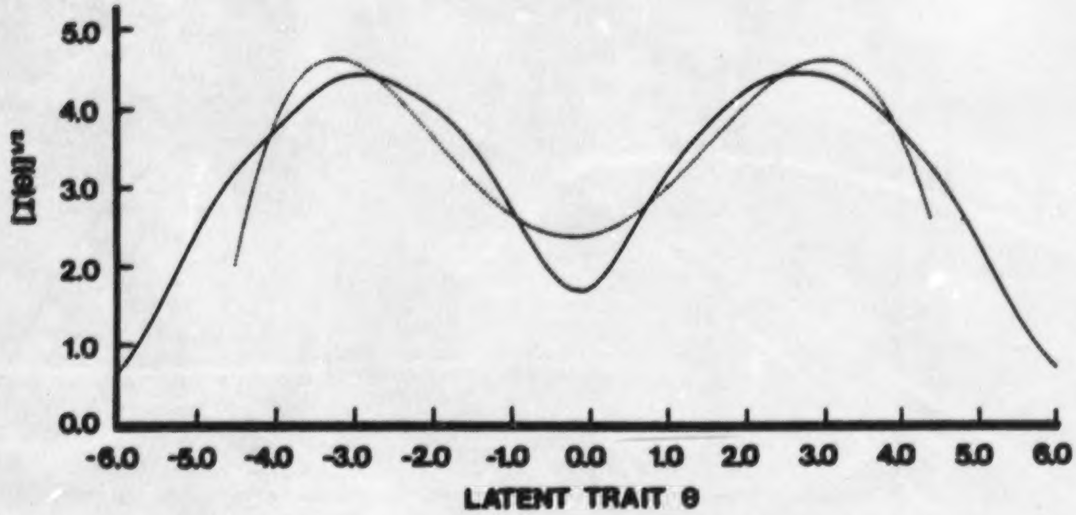
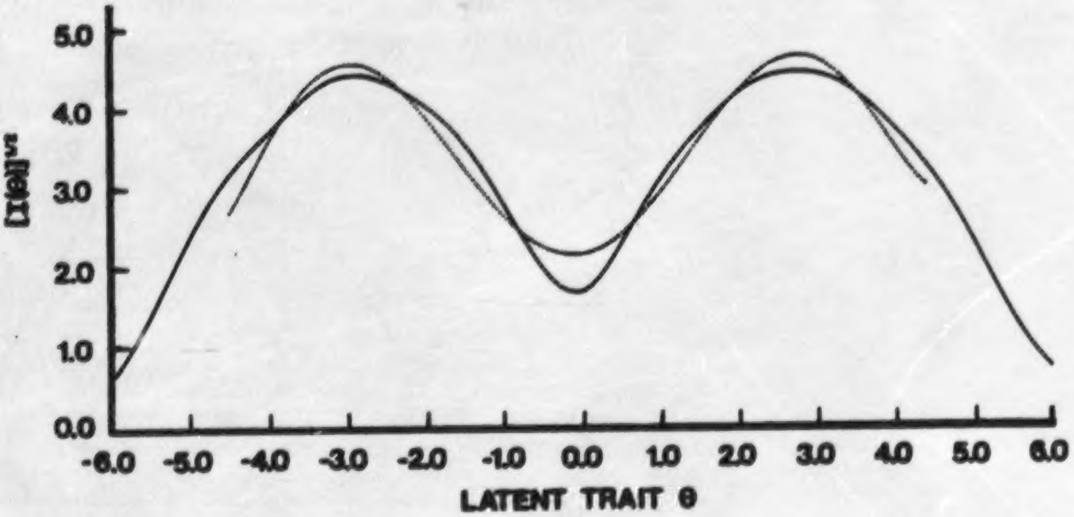
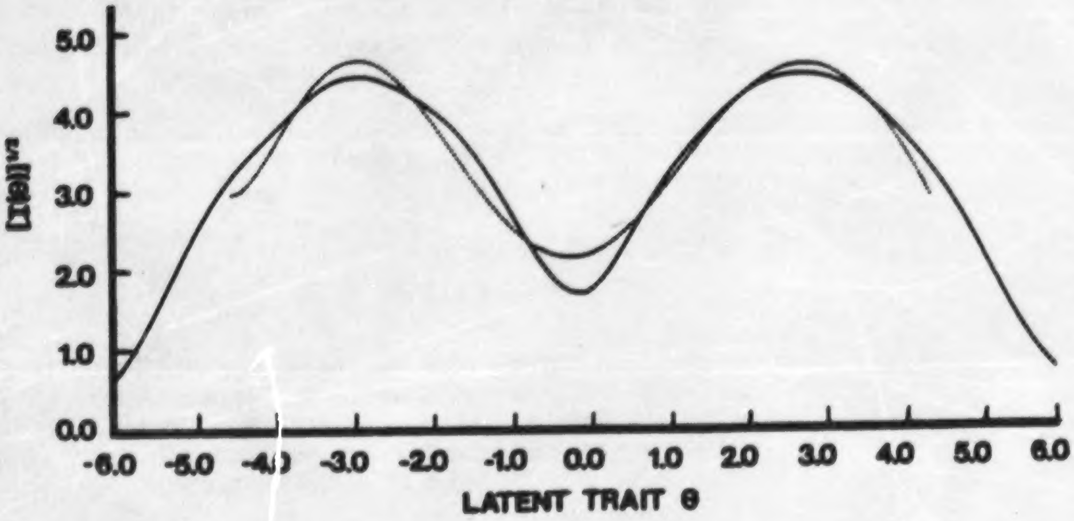


FIGURE A-1-2

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) of Subtest 2 and the Polynomials of Degrees 3 through 7 (Dotted Line), Which Were Fitted by the Method of Moments with $[-4.5, 4.5]$ as the Interval of θ .


5.0
4.0
3.0
2.0
1.0
0.0
[I(θ)]1/2
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ


5.0
4.0
3.0
2.0
1.0
0.0
[I(θ)]1/2
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ


5.0
4.0
3.0
2.0
1.0
0.0
[I(θ)]1/2
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ

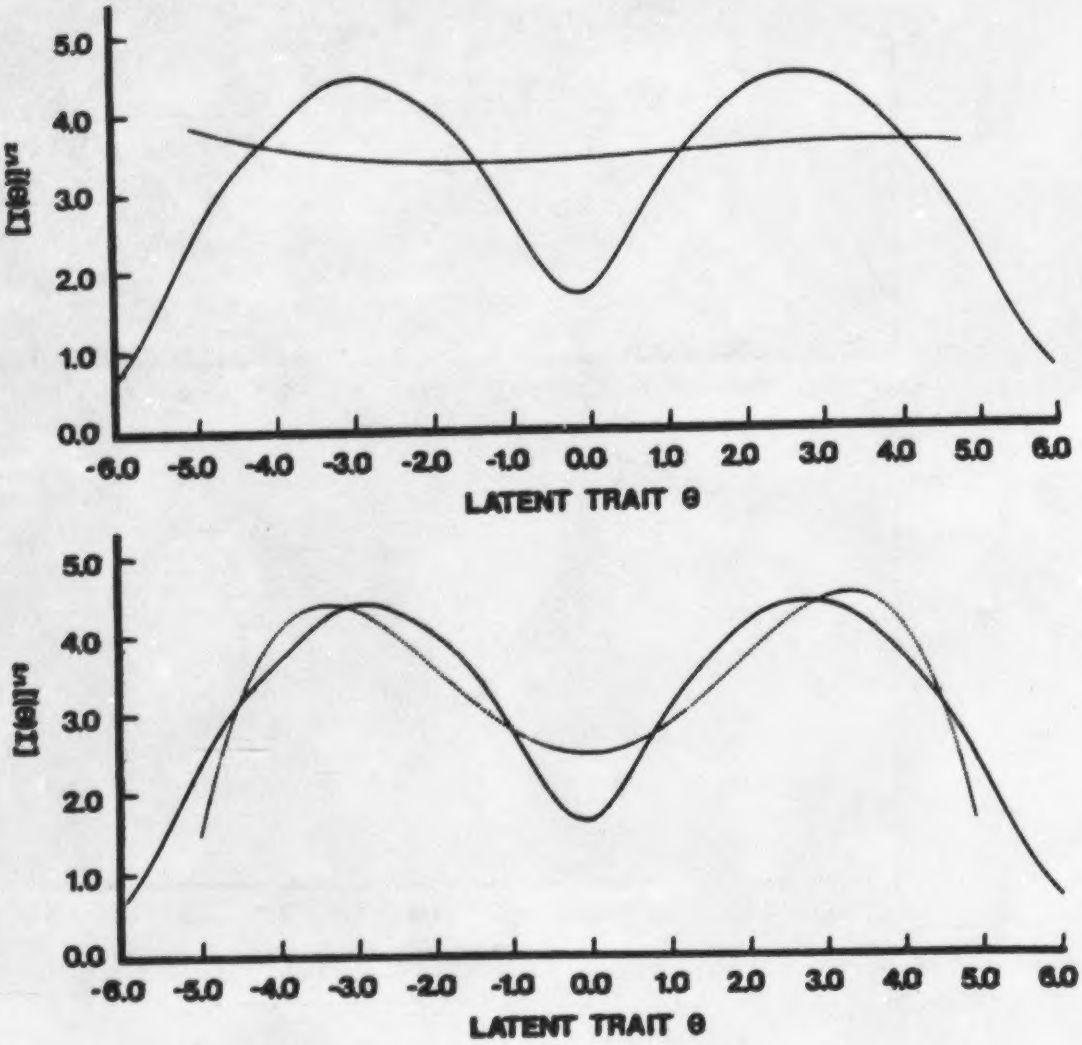
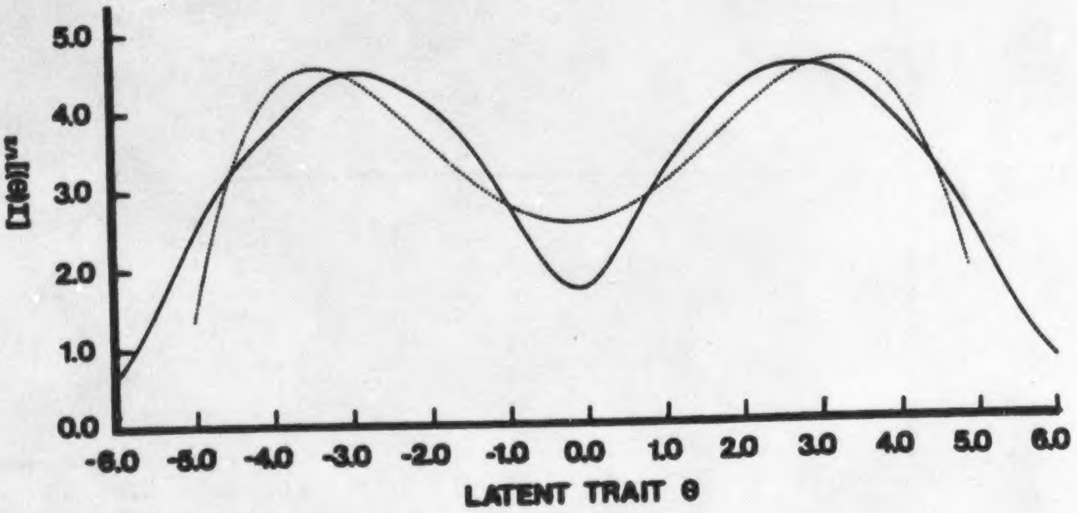
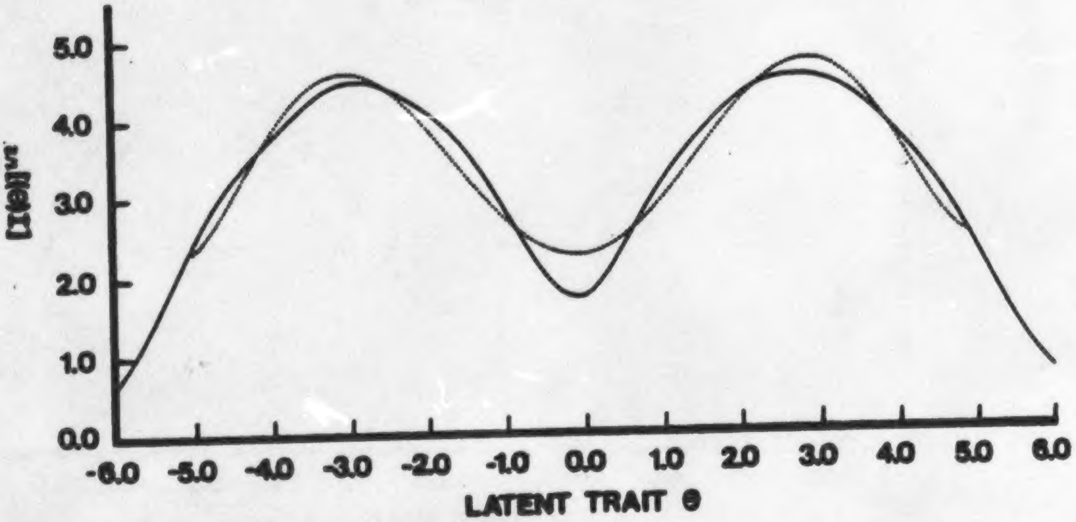


FIGURE A-1-3

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) of Subtest 2 and the Polynomials of Degrees 3 through 7 (Dotted Line), Which Were Fitted by the Method of Moments with [-5.0, 5.0] as the Interval of θ.


[I(θ)]^1/2
5.0
4.0
3.0
2.0
1.0
0.0
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ


[I(θ)]^1/2
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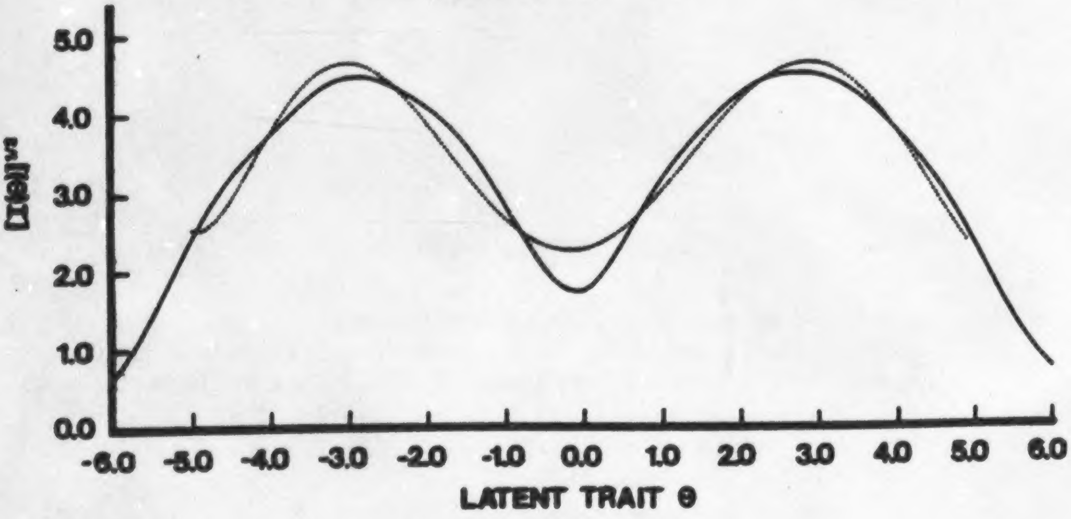
4.0
3.0
2.0
1.0
0.0
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ


[I(θ)]^1/2
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3.0
2.0
1.0
0.0
-6.0
-5.0
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-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ

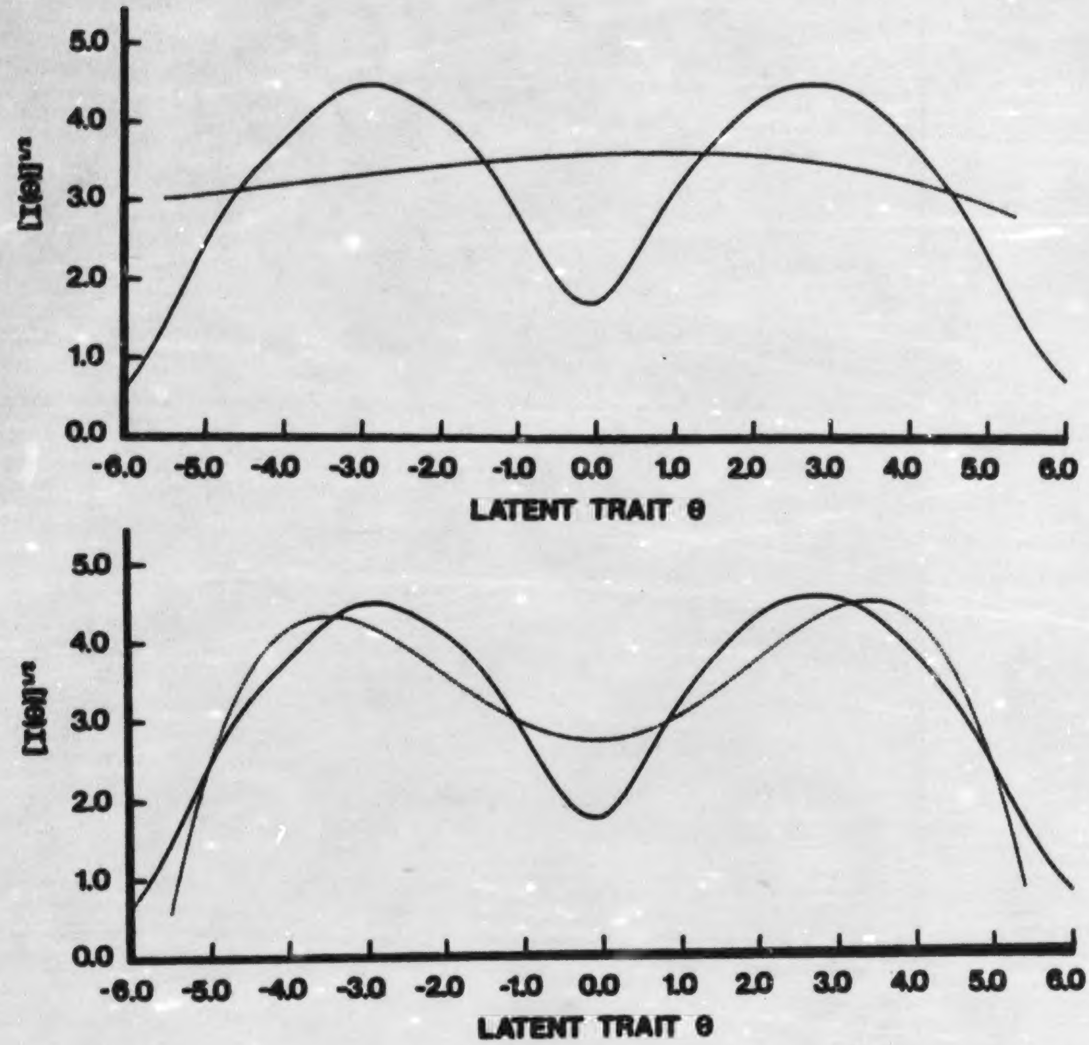
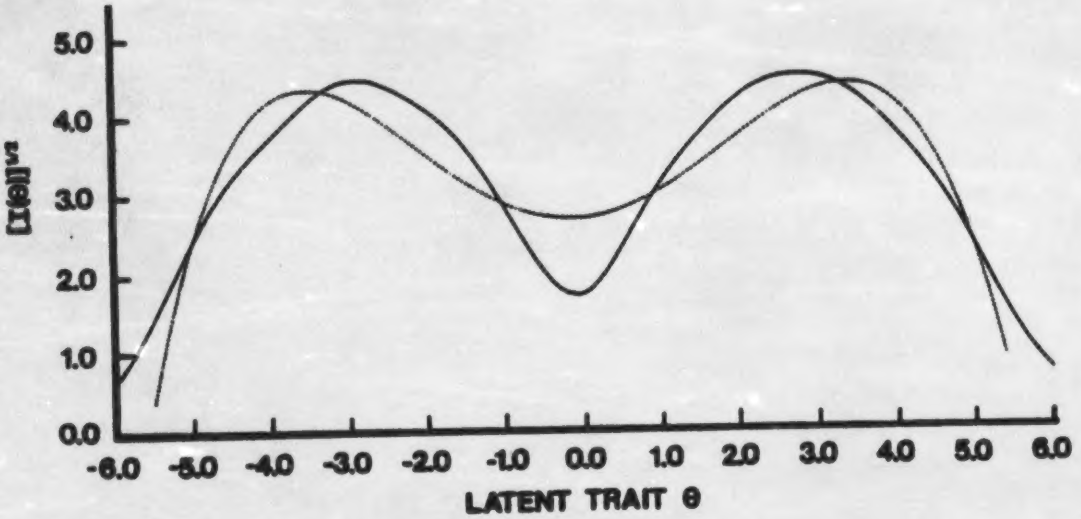
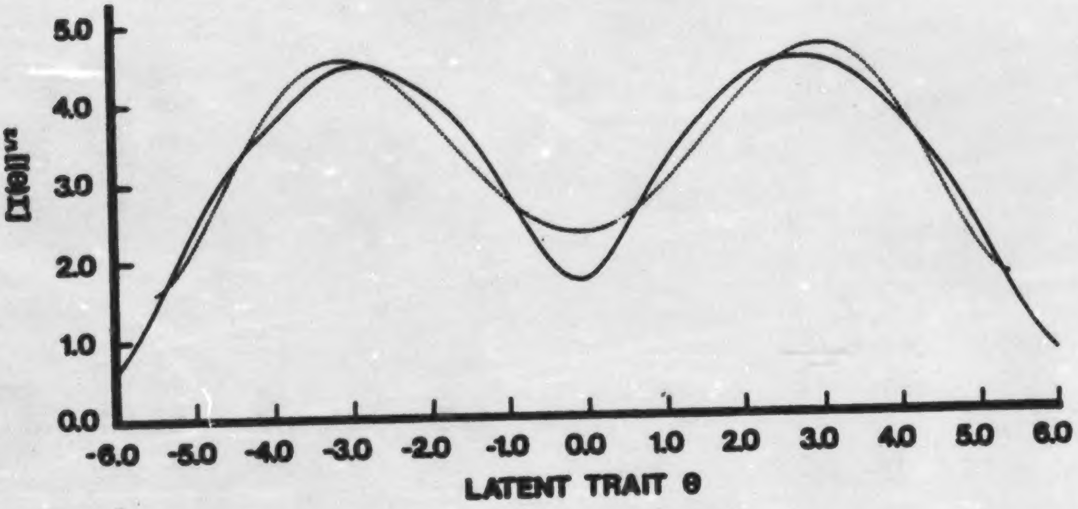
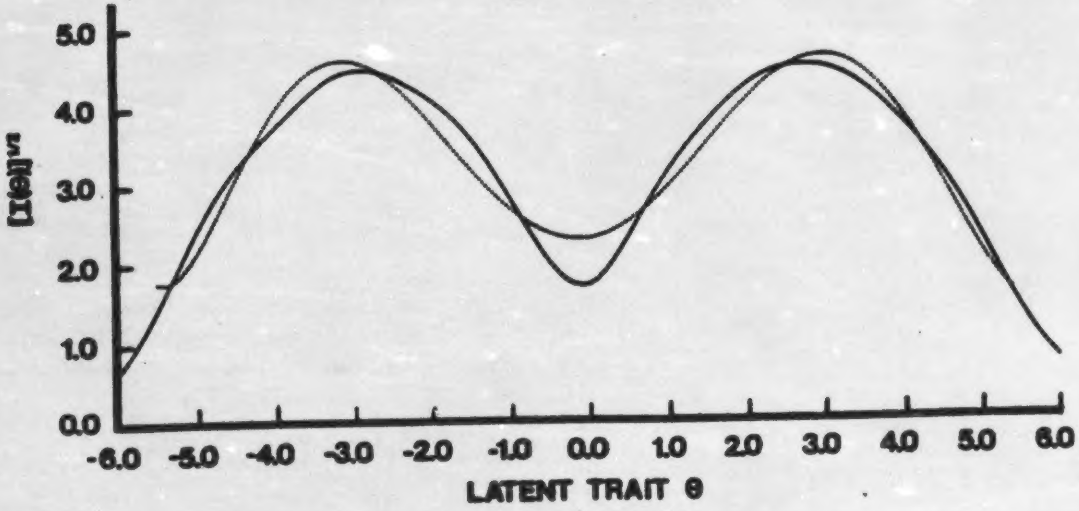


FIGURE A-1-4

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) of Subtest 2 and the Polynomials of Degrees 3 through 7 (Dotted Line), Which Were Fitted by the Method of Moments with [-5.5, 5.5] as the Interval of θ .


5.0
4.0
3.0
2.0
1.0
0.0
[I(θ)]^1/2
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ


5.0
4.0
3.0
2.0
1.0
0.0
[I(θ)]^1/2
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ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4.0
3.0
2.0
1.0
0.0
[I(θ)]^1/2
-6.0
-5.0
-4.0
-3.0
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-1.0
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1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ

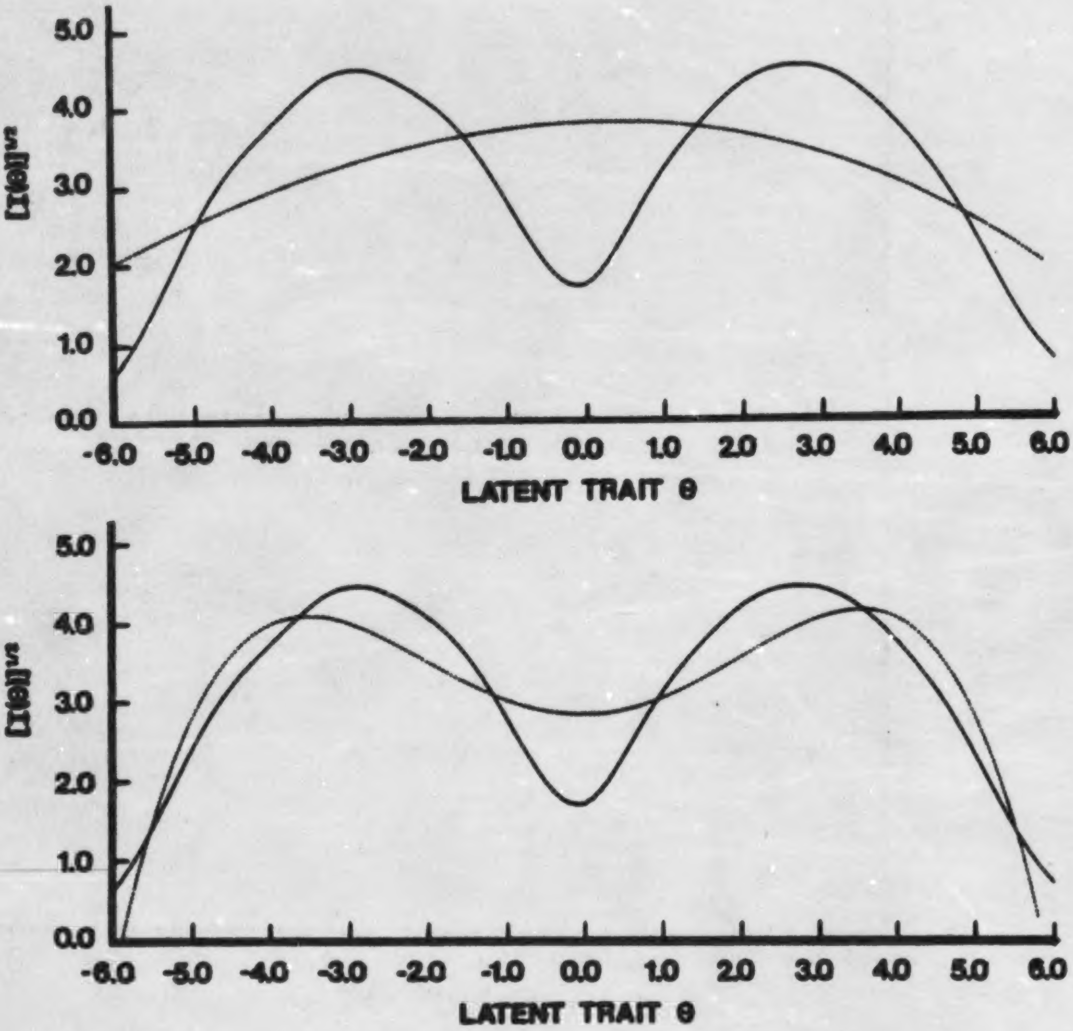
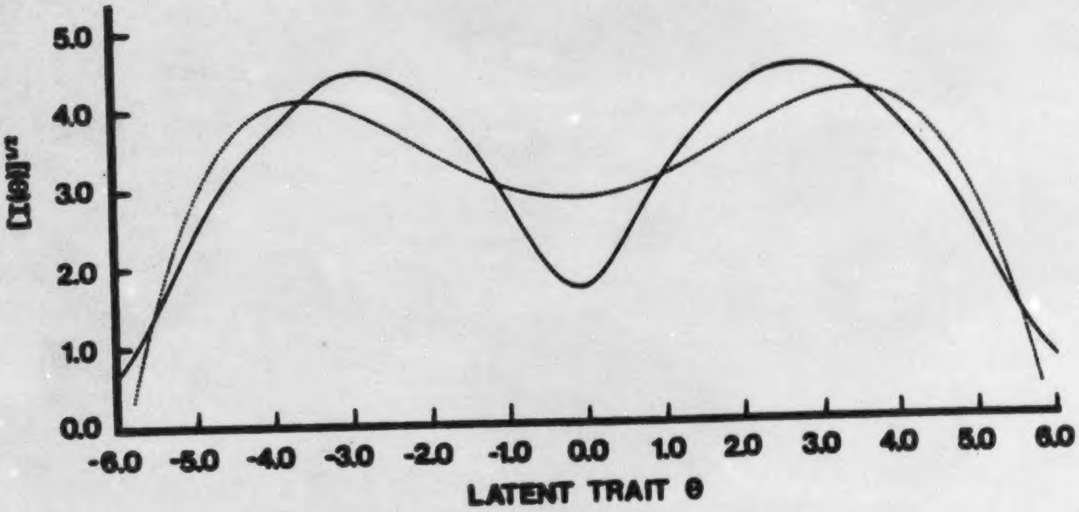
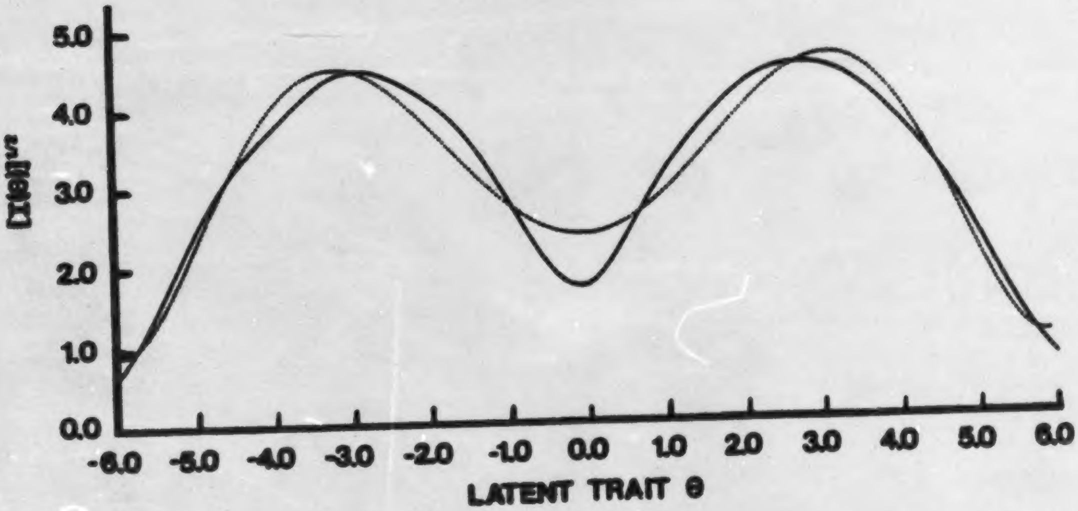
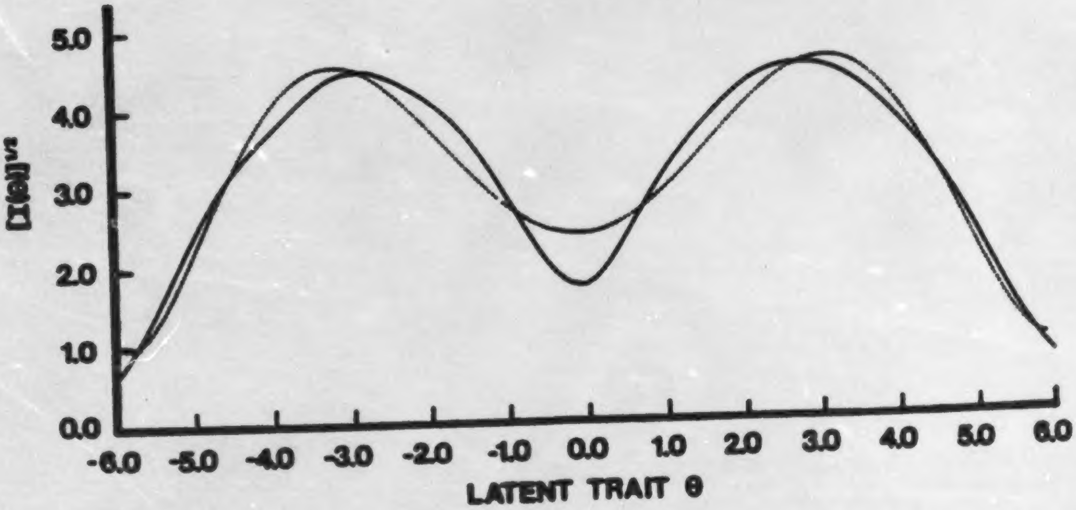


FIGURE A-1-5

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) of Subtest 2 and the Polynomials of Degrees 3 through 7 (Dotted Line), Which Were Fitted by the Method of Moments with [-6.0, 6.0] as the Interval of θ .


[I(θ)]^1/2
5.0
4.0
3.0
2.0
1.0
0.0
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ


[I(θ)]^1/2
5.0
4.0
3.0
2.0
1.0
0.0
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ


[I(θ)]^1/2
5.0
4.0
3.0
2.0
1.0
0.0
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
LATENT TRAIT θ

APPENDIX II

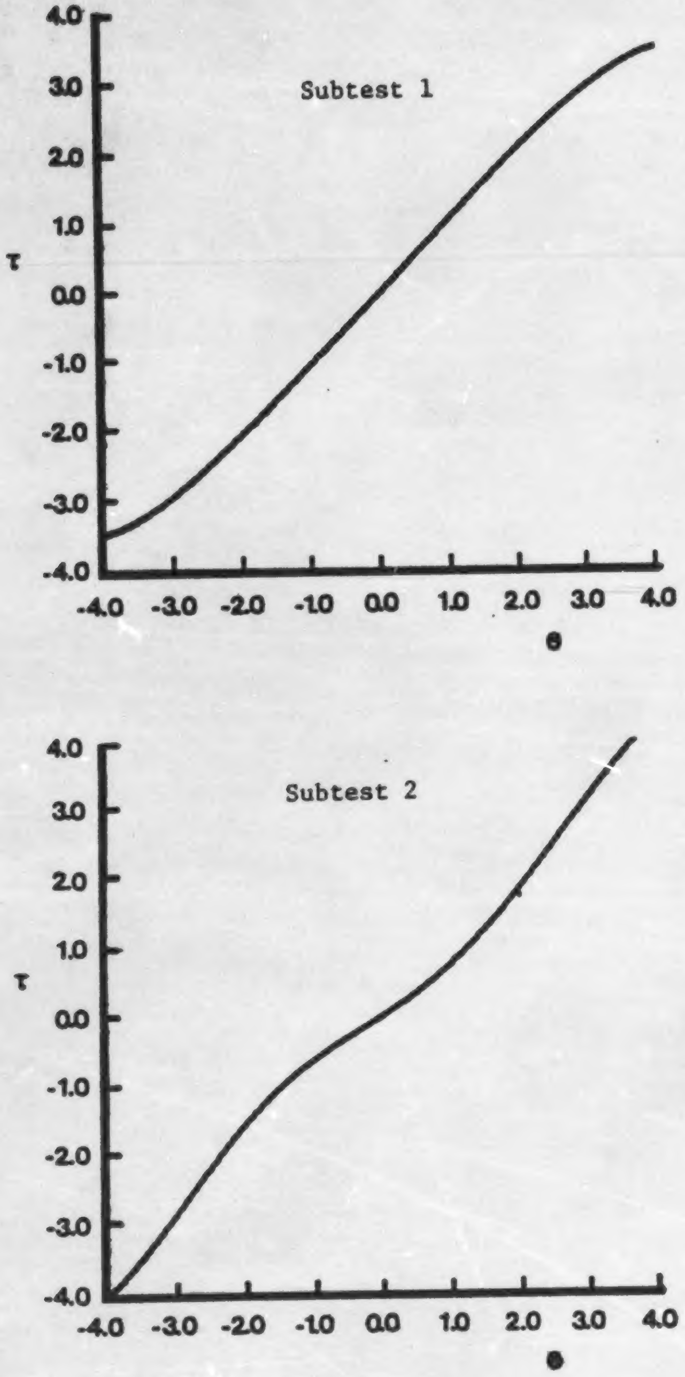


FIGURE A-2-1

Transformation of θ to τ for Subtest 1 (Above) and for Subtest 2 (Below).

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